Analytic Formulas for High-Order Harmonic Generation

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Synopsis We present closed-form analytic formulas for high-order harmonic generation (HHG) rates for both negative ions and neutral atoms. These formulas provide a fully quantum justification for the famous classical three-step scenario (TSS) of HHG as well as a correction to the well-known classical law for the HHG plateau cutoff energy.

For the case of an electron bound in a short-range potential, we present a closed-form analytic formula for high-order harmonic generation (HHG) rates (obtained in the quasiclassical limit [1]) that shows excellent agreement with exact time-dependent effective range results over the high energy part of the HHG plateau (and beyond). It justifies the empirical parametrization of HHG rates suggested in Refs. [2]. Our analytic results allow one to describe the dependence of the oscillatory patterns of HHG rates on both the harmonic number N and on the laser parameters (for a given N), as well as the dependence of the HHG rates on the orbital angular momentum, l, of the bound electron. Our analytic formula for HHG rates can be presented as the product of three factors, which have clear physical meanings within the TSS:

$$R_N = I(F_0)W(E)\sigma(E), \quad E = N\hbar\omega - |E_0|. \quad (1)$$

$I(F_0)$ describes the first (*ionization*) step in the TSS: it is proportional to the detachment rate in a static electric field, whose strength F_0 is that of the laser electric field at the moment of tunneling. $W(E)$ describes the second step, the *propagation* of the detached electron in the laser field from the moment of tunneling up to the recombination event. This factor is essentially independent of the shape of the atomic potential. It involves a single Airy function that describes interference of two ("short" and "long") classical electron trajectories and depends on the difference $E - E_{max}$, where E_{max} is the maximum kinetic energy gained by the active electron from the laser field. The photorecombination cross section $\sigma(E)$ describes the final step in the TSS: the *recombination* of the active electron with energy E to the initial bound state of energy E_0 with emission of the harmonic photon. Note that an HHG rate involving a combination of the Airy function and its first derivative was obtained previously in Ref. [3] for a zero-range potential model within the uniform approximation; however, the arguments of these functions were not expressed in closed analytic form.

Our analytic result (1) was obtained in Ref. [1] for the case of an electron in a short-range potential (negative ion). However, as each of the three factors in (1) has a transparent physical meaning within the TSS (which, as is commonly accepted, does not depend on the atomic species), one can expect that Eq. (1) can be generalized appropriately to describe HHG for atoms. This generalization consists in the replacement of the ionization and recombination factors, $I(F_0)$ and $\sigma(E)$, by their atomic counterparts. We show that the resulting Coulomb-modified expression (1) gives results for the high energy part of the HHG spectrum for the H atom that are in perfect agreement with those obtained by numerical solution of the time-dependent Schrdinger equation.

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References

[1] M.V. Frolov, N.L. Manakov, T.S. Sarantseva and A.F. Starace, J. Phys. B **42**, 035601 (2009).

[2] J. Itatani et al., Nature (London) **432**, 867 (2004); T. Morishita, A.T. Le, Z. Chen and C.D. Lin, Phys. Rev. Lett. **100**, 013903 (2008).

[3] D. B. Milošević and W. Becker, Phys. Rev. A **66**, 063417 (2002).

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