ENHANCING THE OBSERVABILITY OF THE *Efimov Effect* IN ULTRACOLD ATOMIC GAS MIXTURES

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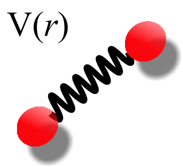
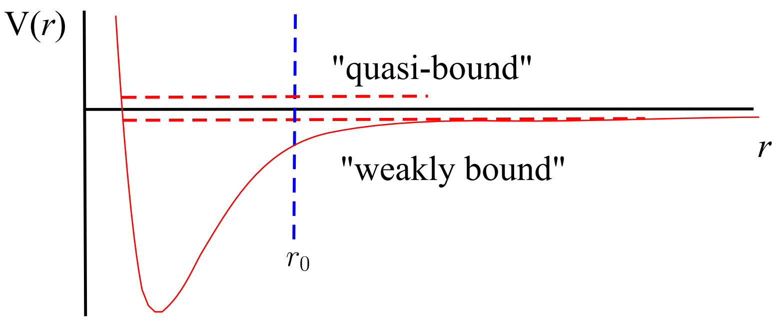
National Science Foundation

What is this Efimov Effect?

EFIMOV EFFECT

Vitaly Efimov, "Energy levels arising from resonant two-body forces in a three-body system", Phys. Lett. **33**, 563 (1970)





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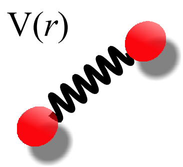

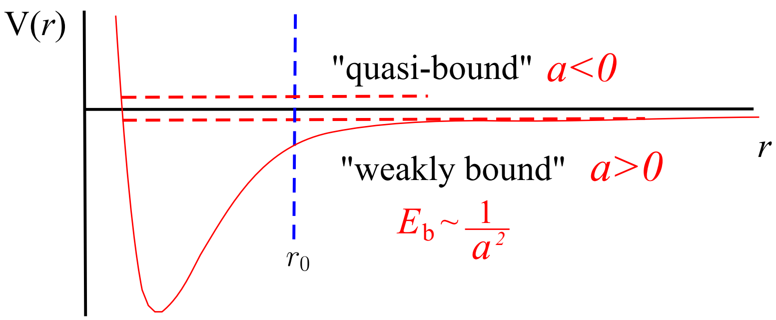

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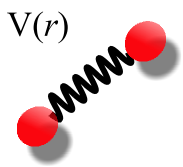
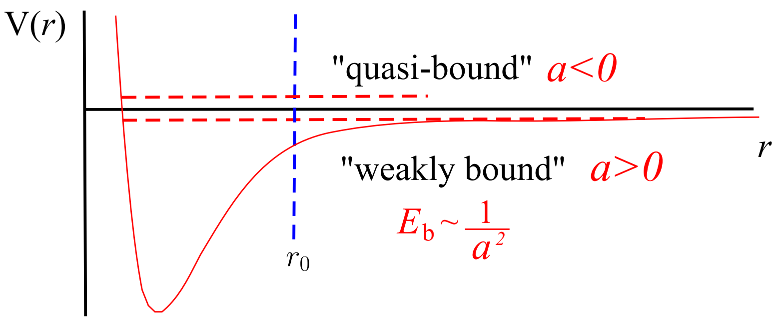


Resonant Interaction

$$|a| \gg r_0$$



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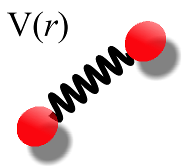

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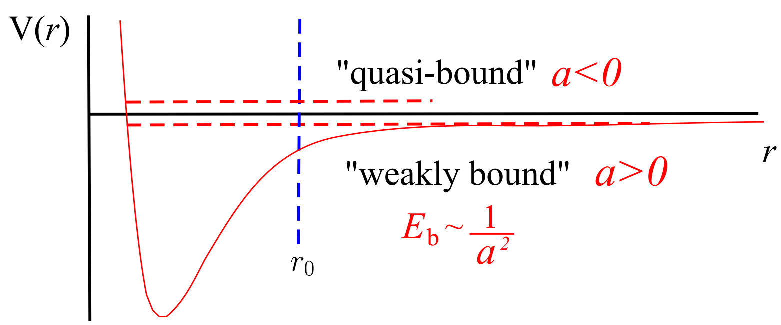
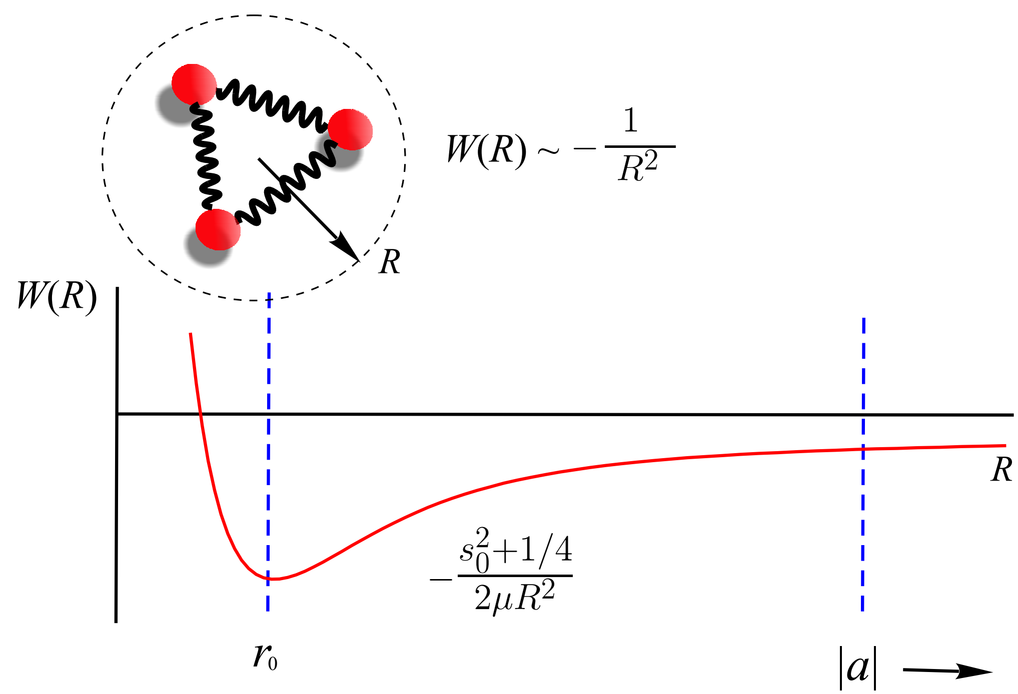


$W(R) \sim -\frac{1}{R^2}$



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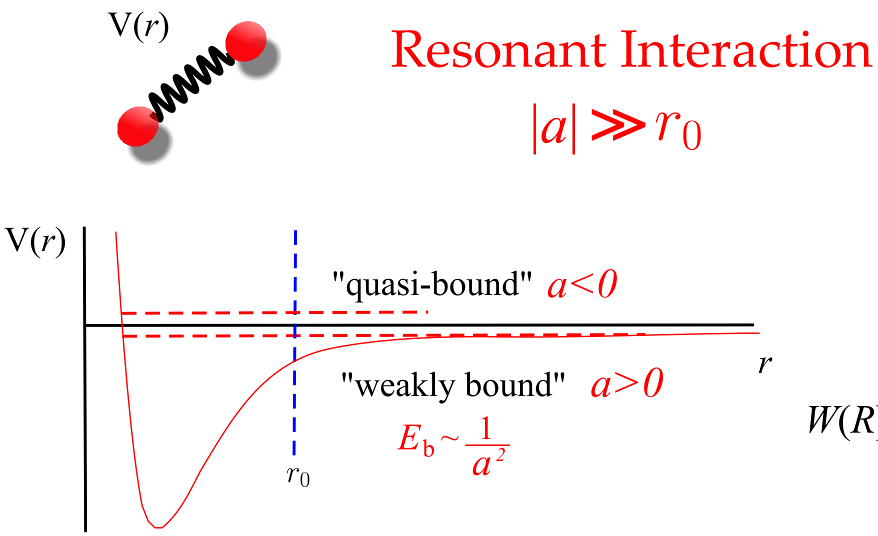

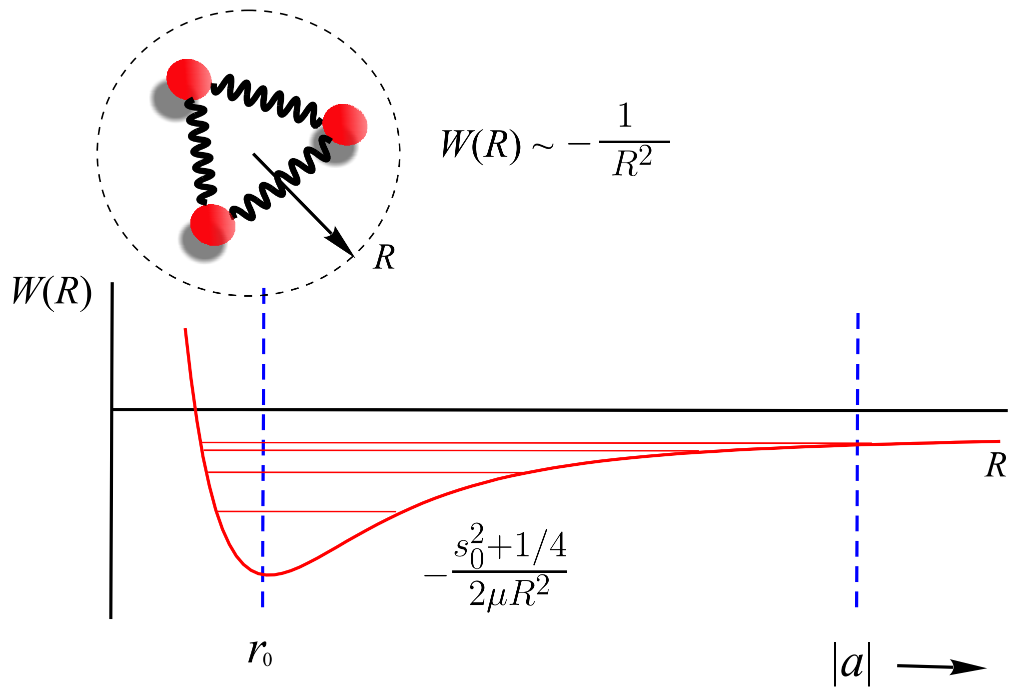

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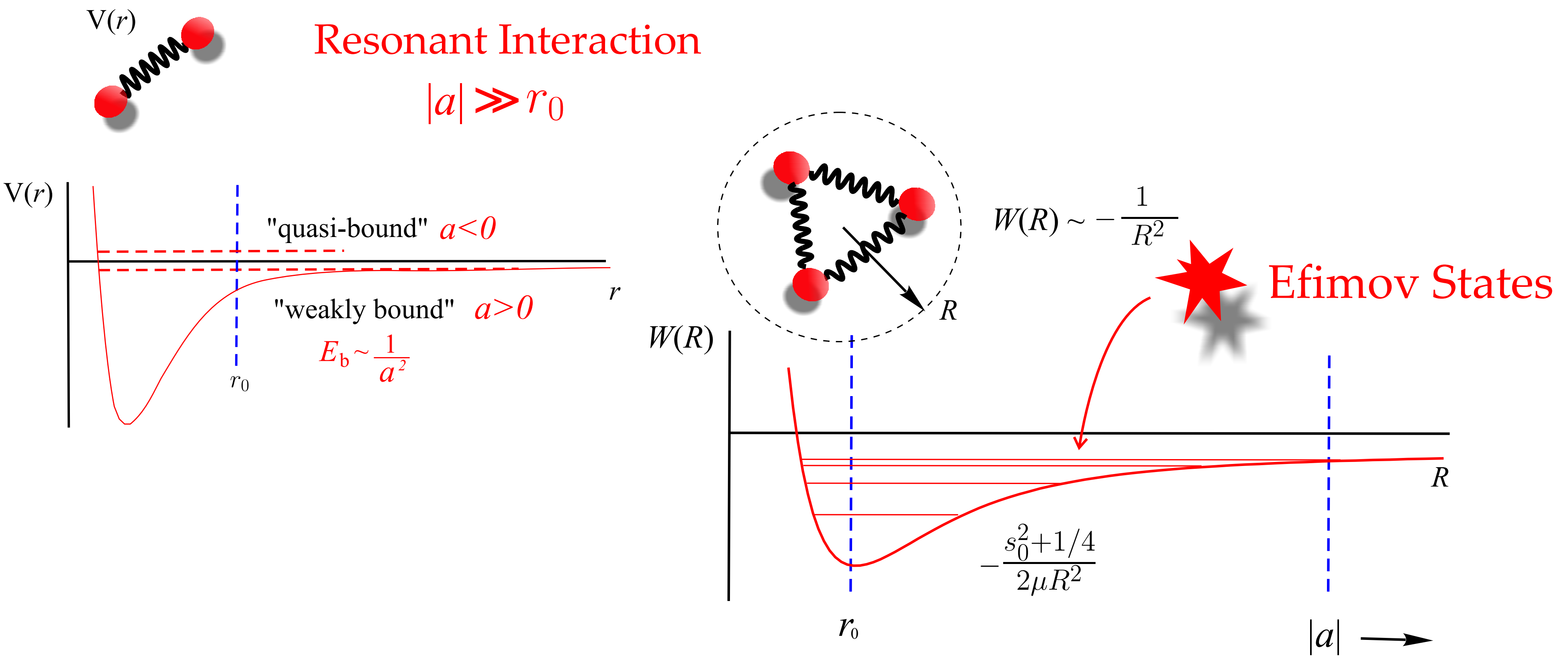

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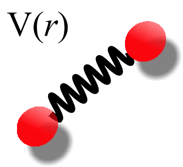

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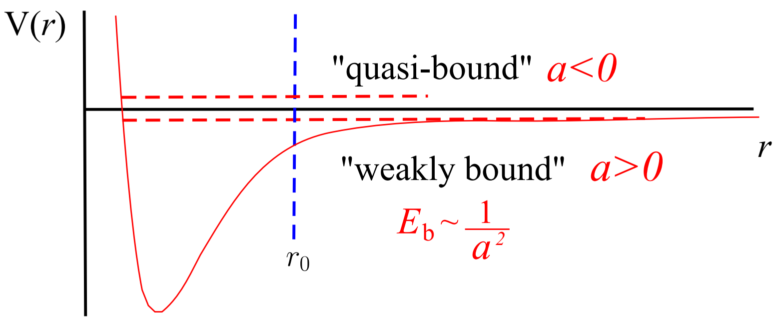
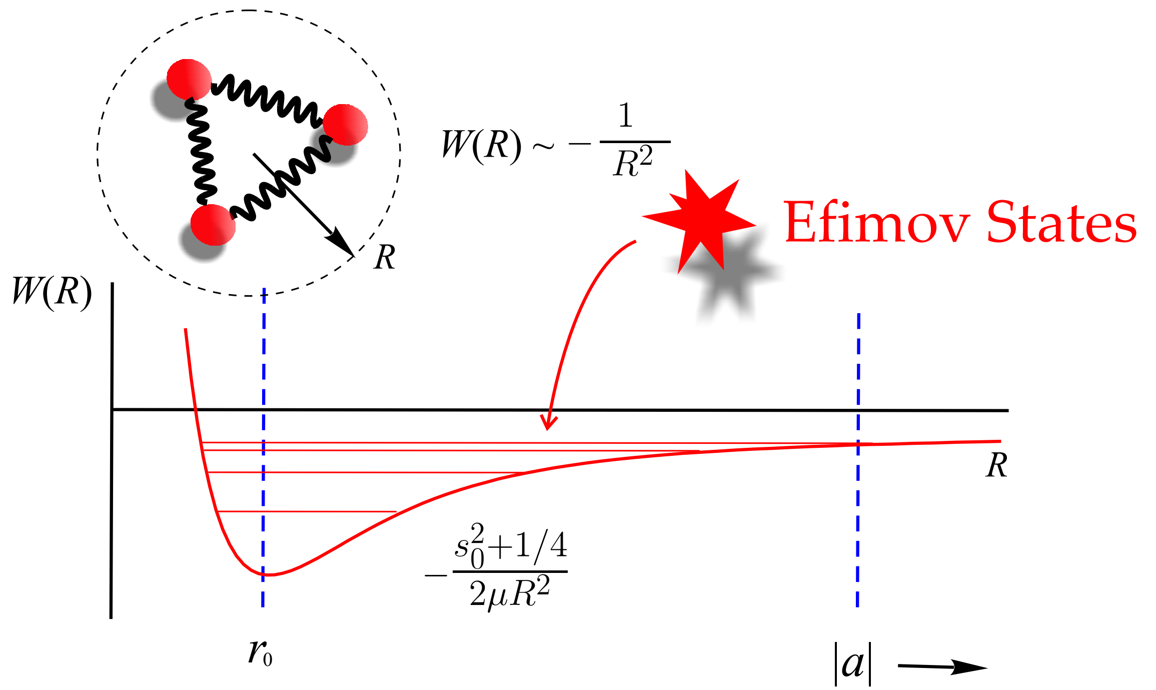




Finite Number of
2-Body bound states
(short-range interaction)

Large Number of
3-Body bound states

Efimov Effect

Search for Efimov States: Nuclear Physics, Atomic Physics

Limitations: needs to find a system with large a

(at least 3 Efimov states)

$\rightarrow$ He_3: only 1 Efimov state: open question !

$\rightarrow$ extremely weakly-bound states !

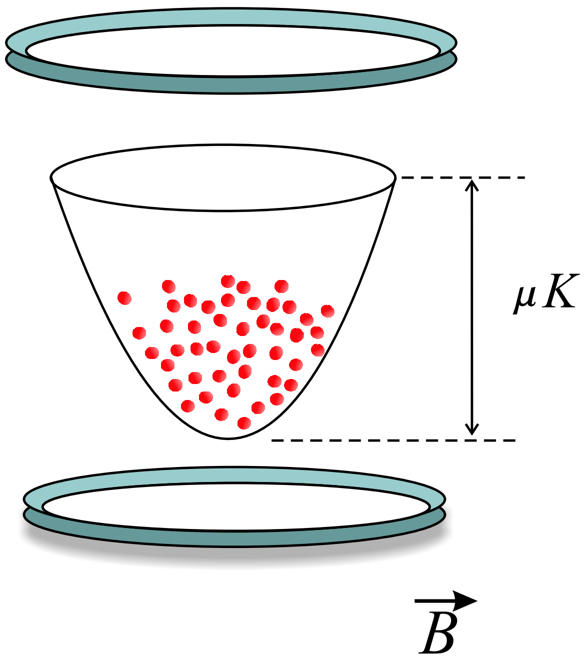
Ultracold Quantum Gases

- Experimentally accessible !
- Clear signature of the Efimov Effect !

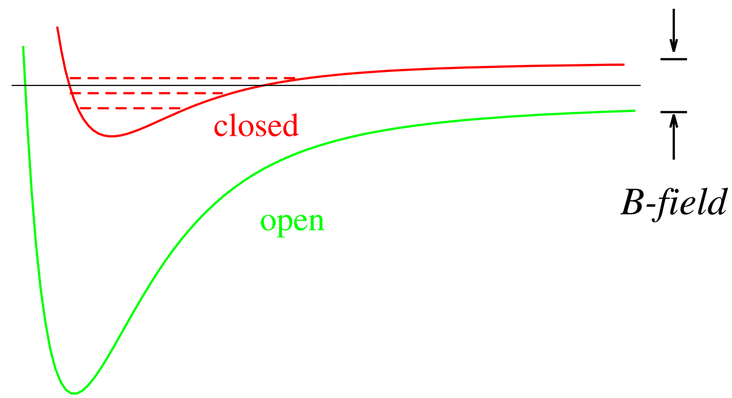
(3-body collisions)

ULTRACOLD QUANTUM GASES

... after evaporative cooling ...



Feshbach Resonances



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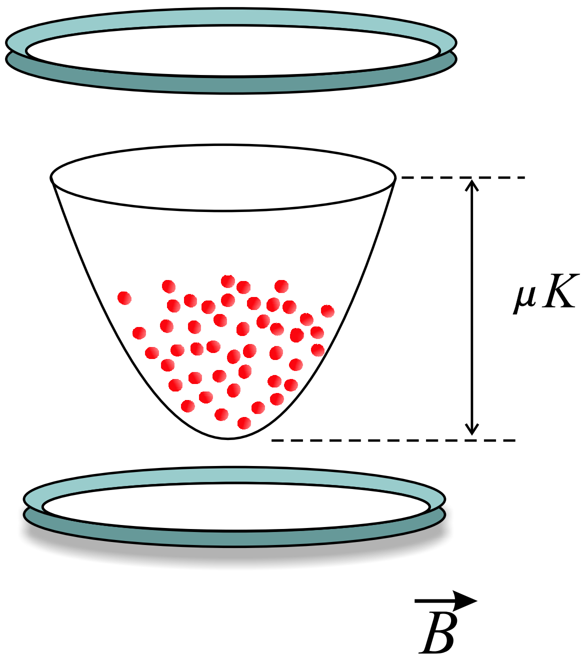

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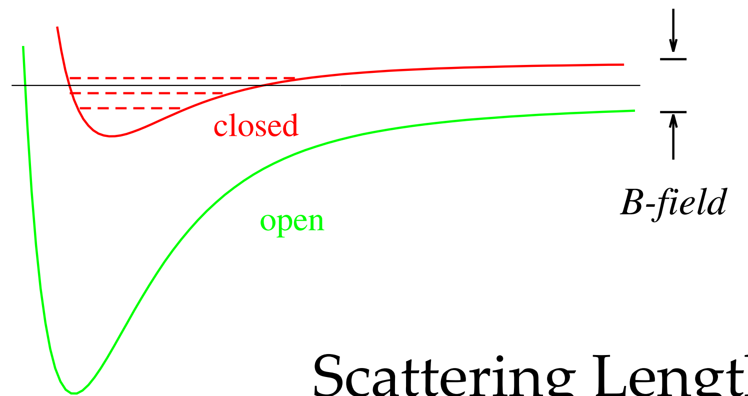
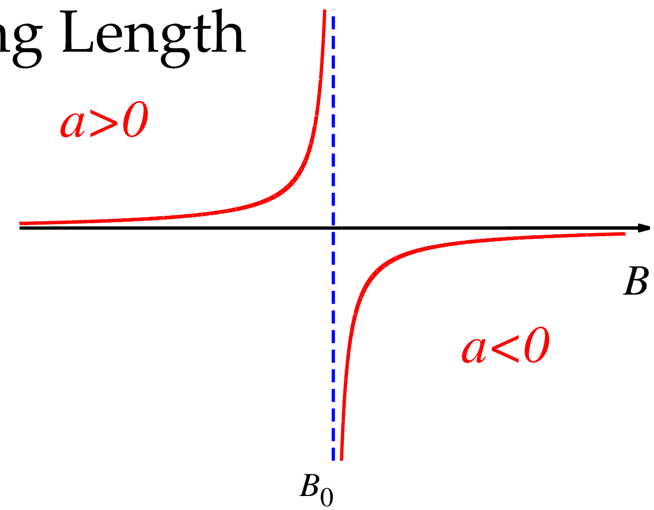


Scattering Length



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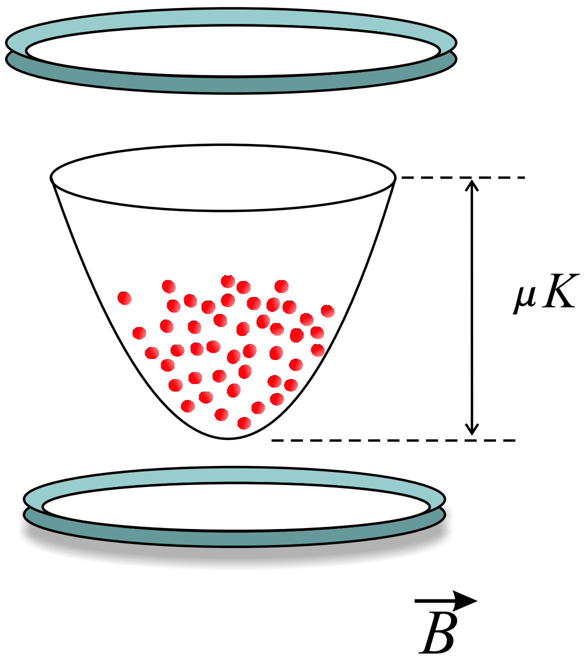

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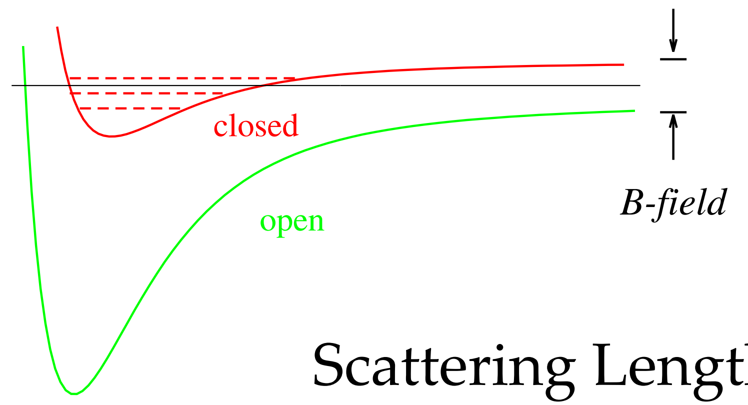

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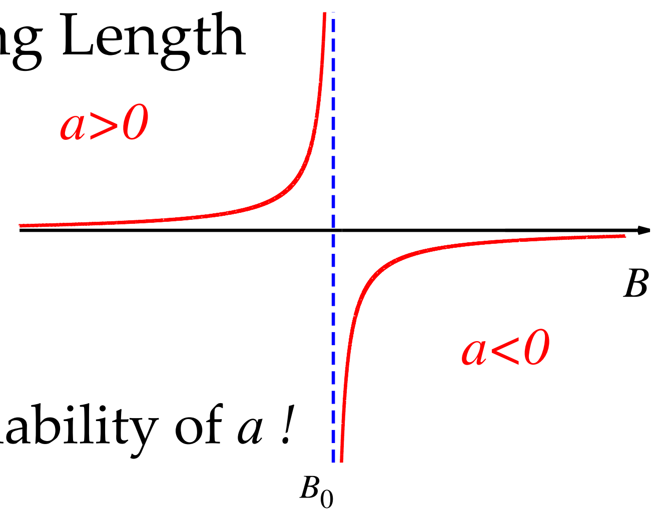

Tunability of *a* !

ULTRACOLD QUANTUM GASES

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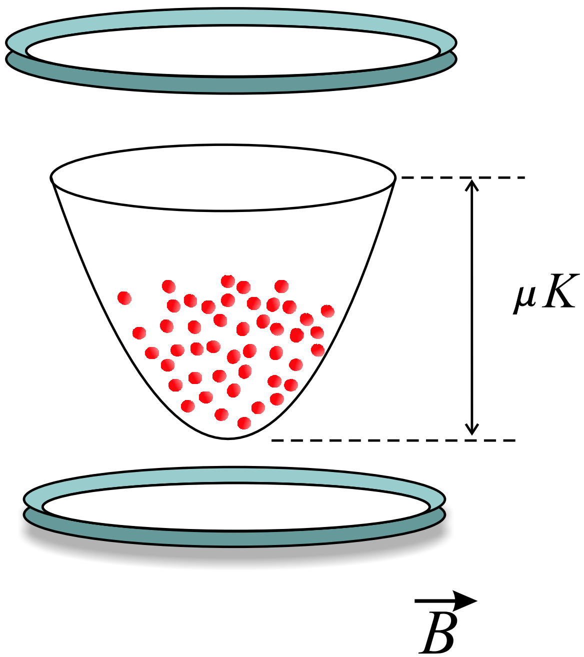

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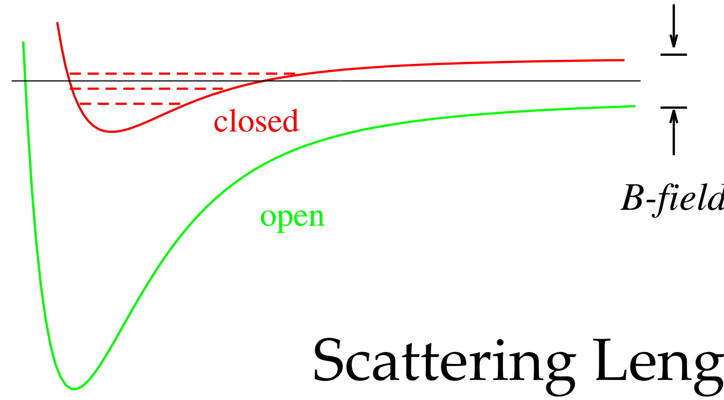

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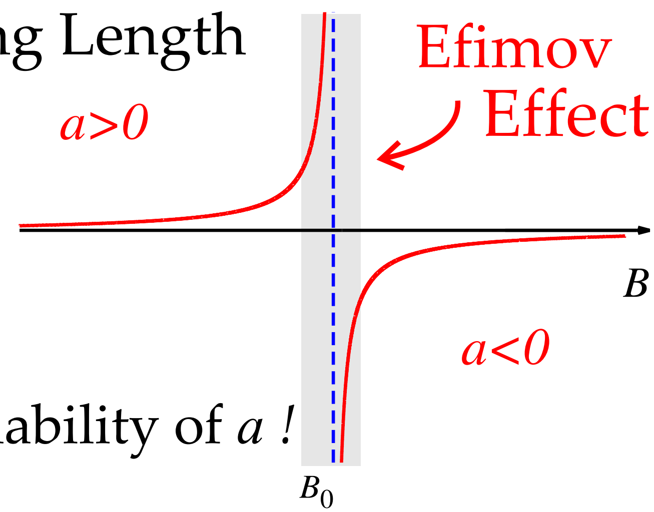

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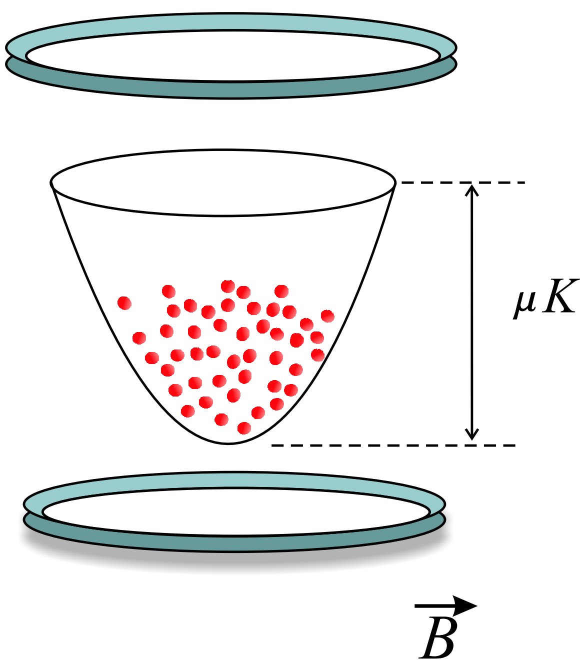

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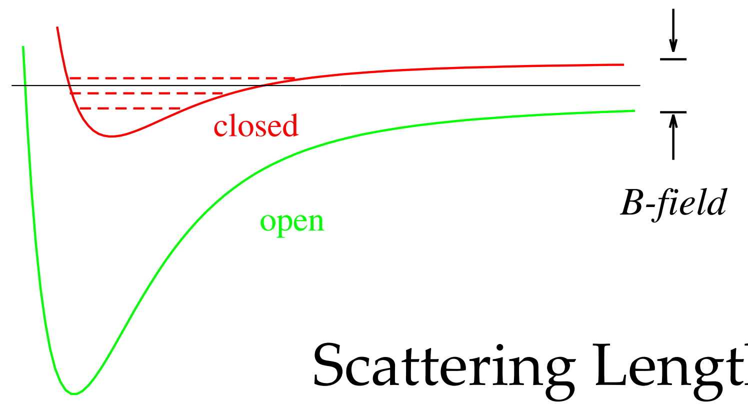

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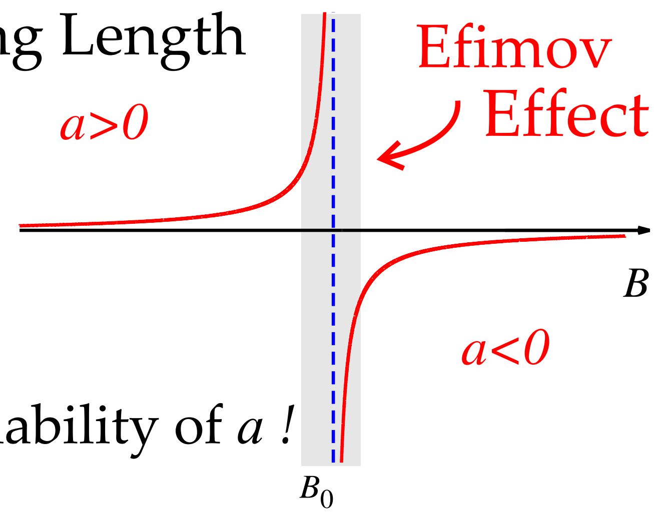
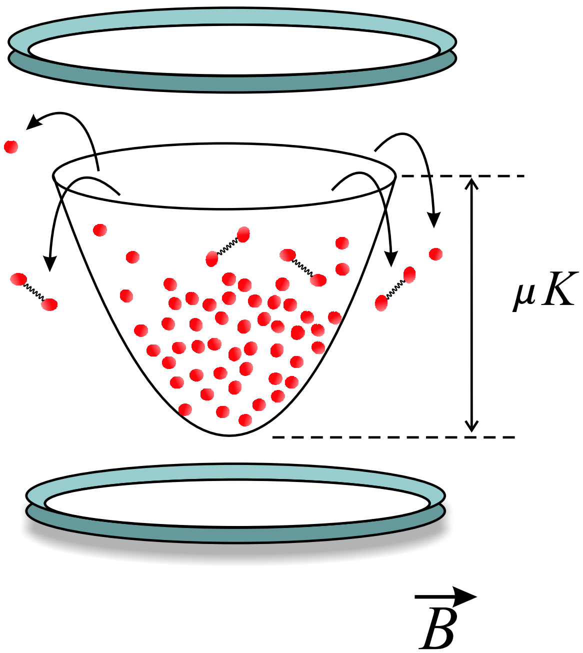


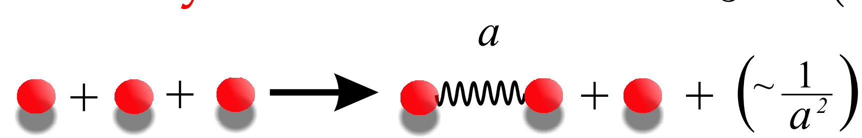
Tunability of a !

2-Body Collisions → Suppressed !

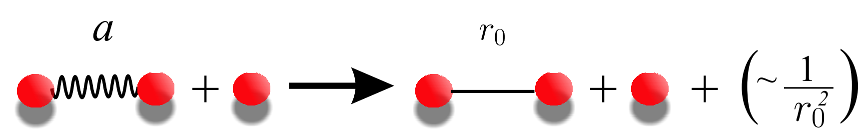
3-Body Collisions → Lifetime, Stability ... Efimov Effect !



Three-Body Recombination: $K_3 \propto (k/\mu)\sigma$



Vibrational Relaxation: $V_{\rm rel} \propto (k/\mu)\sigma$



Rate Equations:

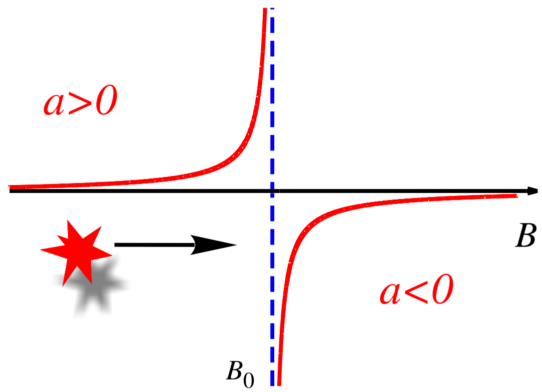
$$\dot{n}_X(t) = -[K_3(a)]n_X^3 - [V_{\rm rel}(a)]n_X n_{X_2}$$

$$\dot{n}_{X_2}(t) = -[V_{\rm rel}(a)]n_X n_{X_2}$$

n_X, n_{X_2}: experimental observables !

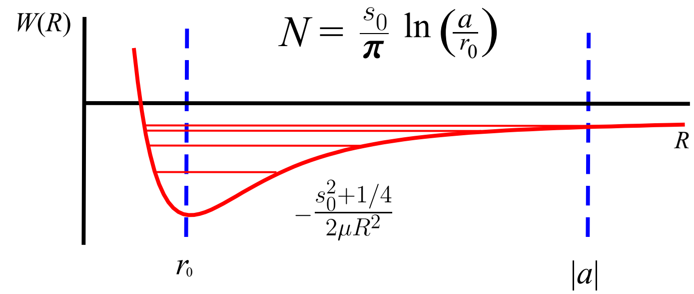
Signatures of Efimov Effect?!

Building Intuitive Picture



Increasing a ...

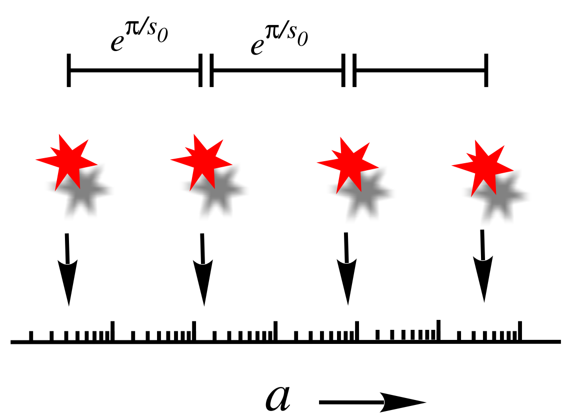
... we are creating new
Efimov States !



New Efimov State: $a \longrightarrow e^{\pi/s_0}$

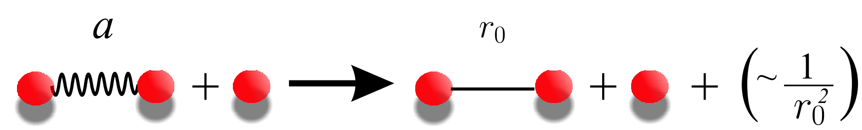
Phase shift jumps of π

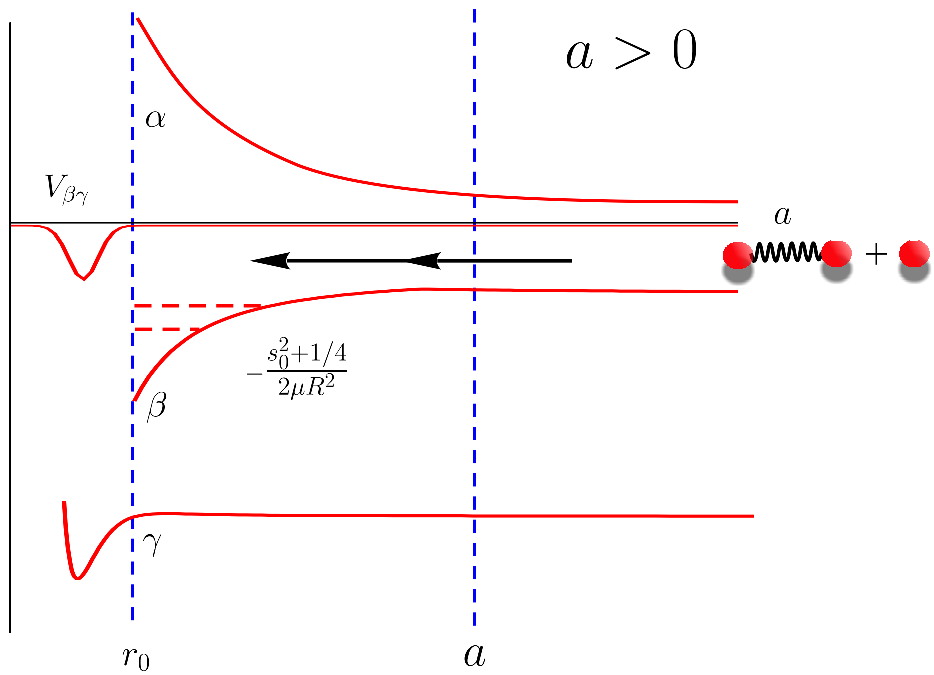
Cross section: Efimov features !



Signatures of The Efimov Effect

<u>Vibrational Relaxation</u>





SIGNATURES OF THE EFIMOV EFFECT

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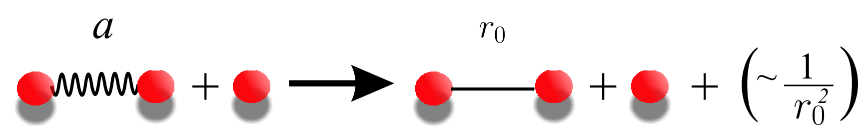

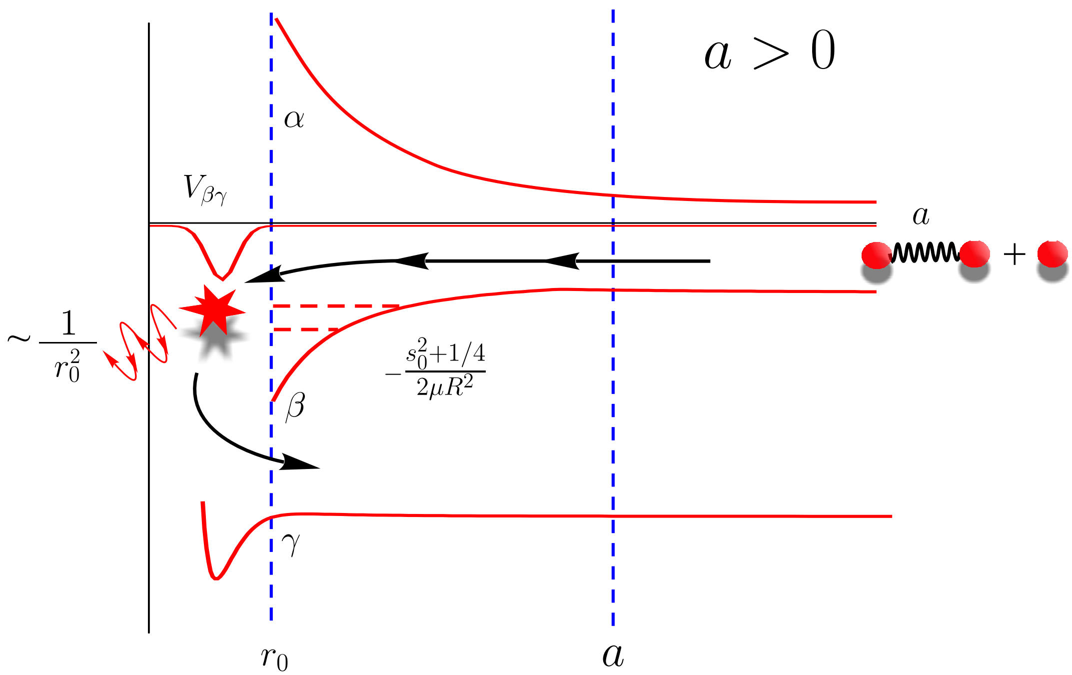

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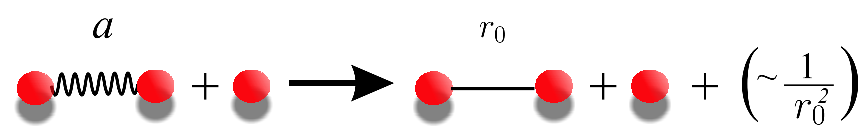

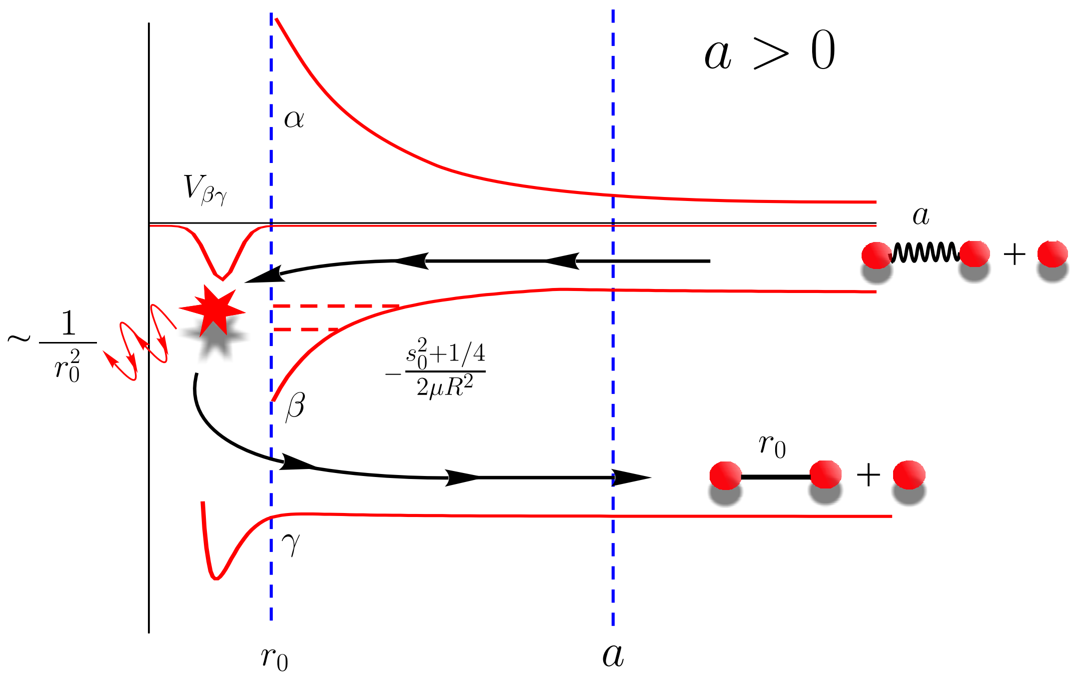

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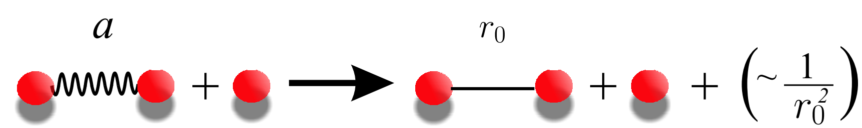

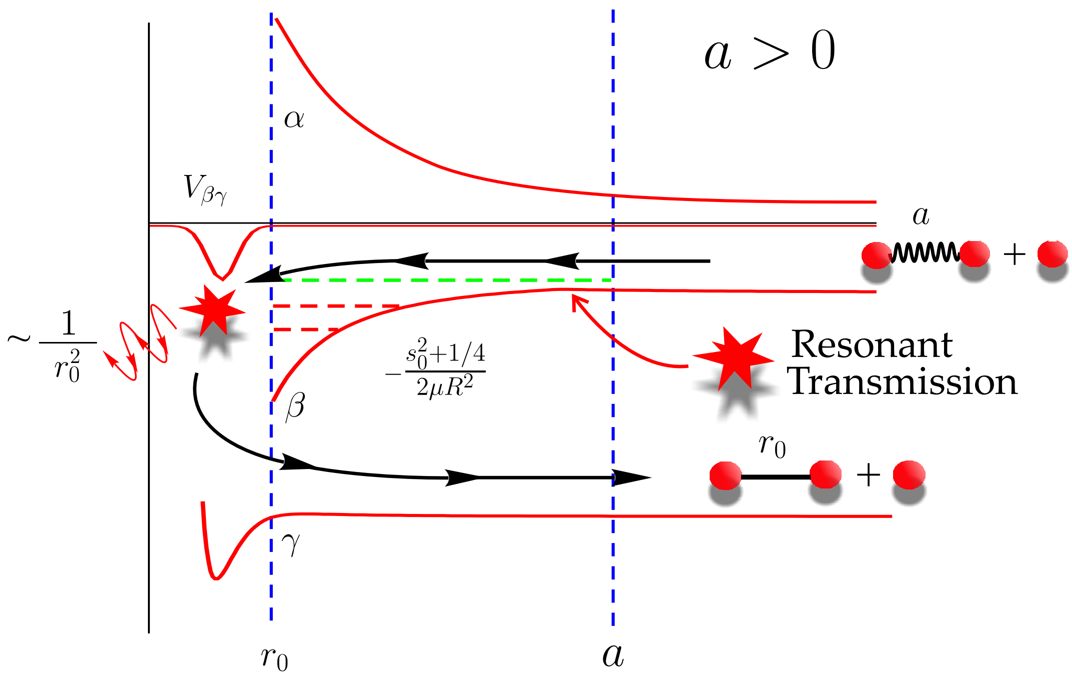

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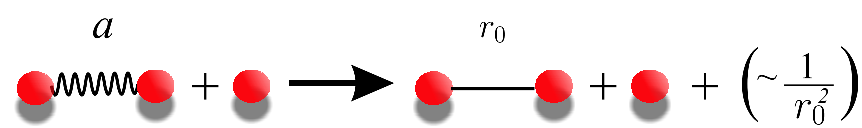

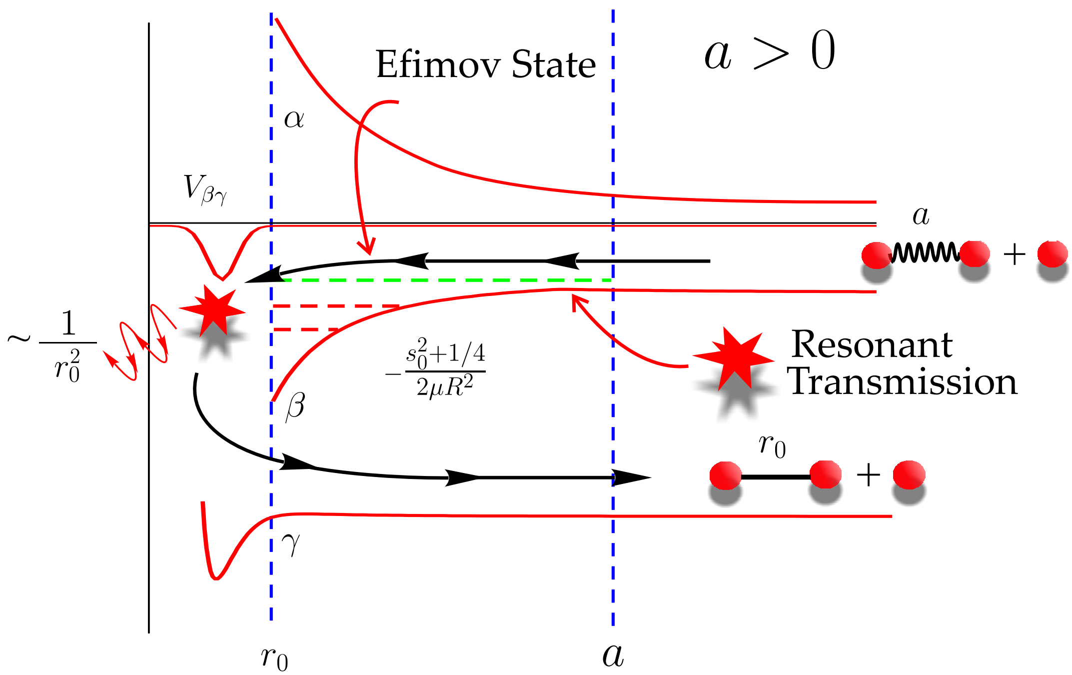

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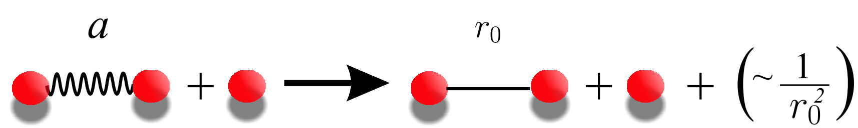
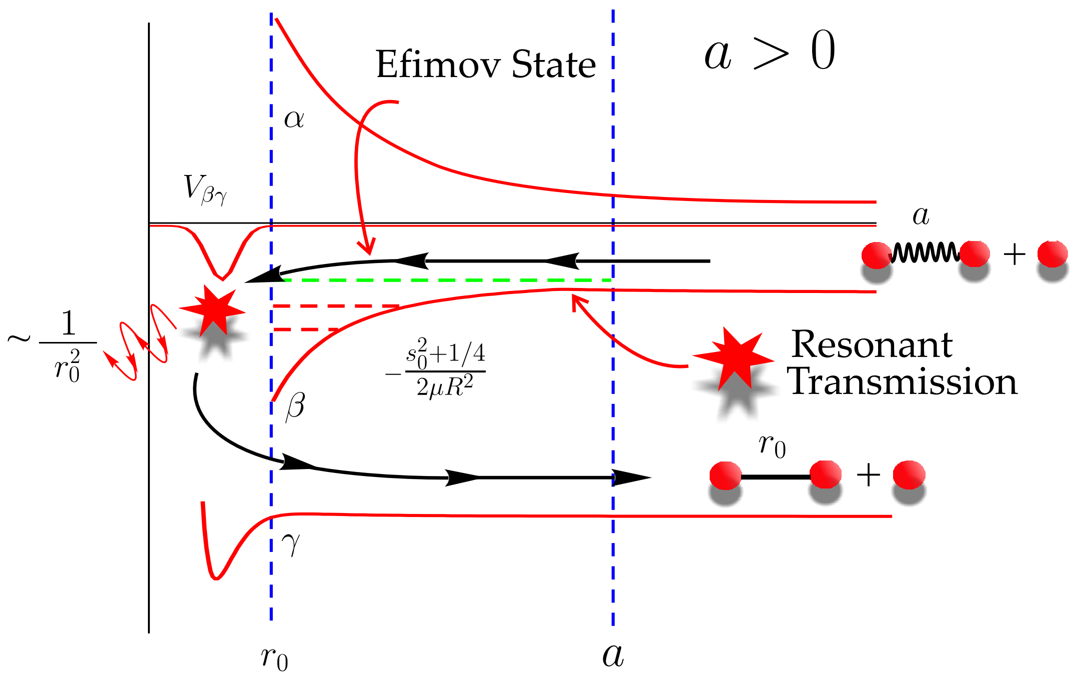


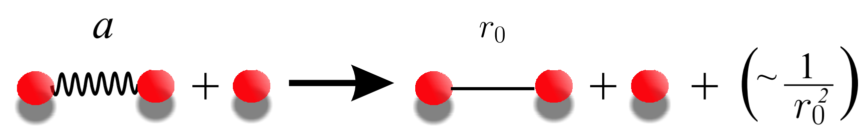


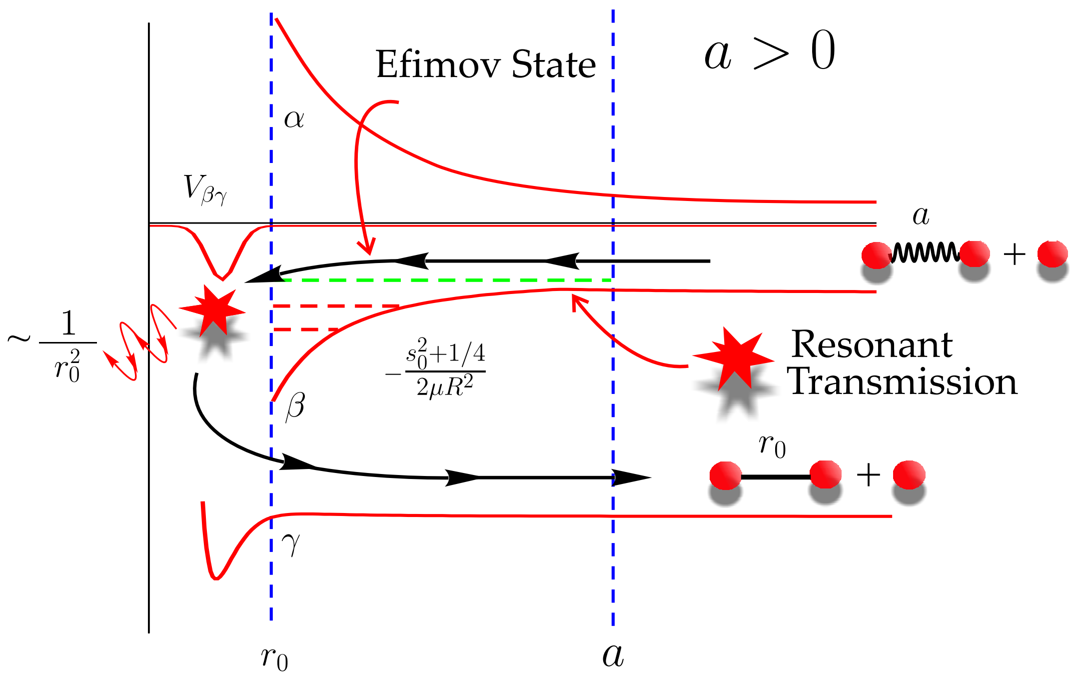
$$V_{\rm rel} = A_\eta \frac{\sinh(2\eta)}{\sin^2\left[s_0 \ln\left(\frac{a}{r_0}\right)+\Phi\right]+\sinh^2(\eta)}\, a$$

A_η, Φ, η: details !

SIGNATURES OF THE EFIMOV EFFECT

Vibrational Relaxation





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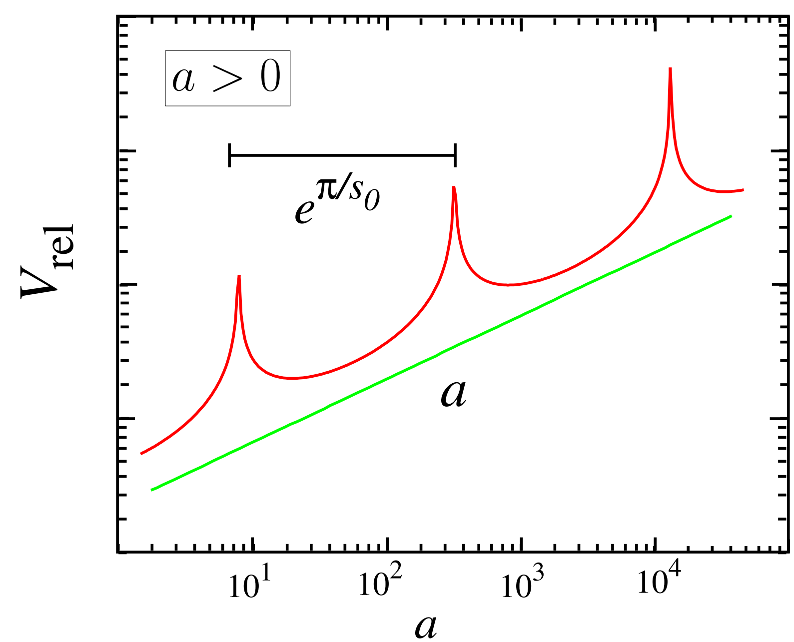
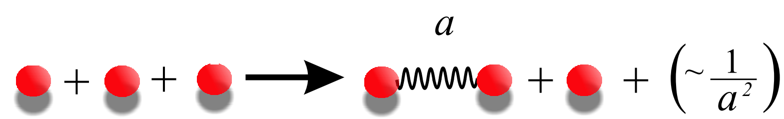
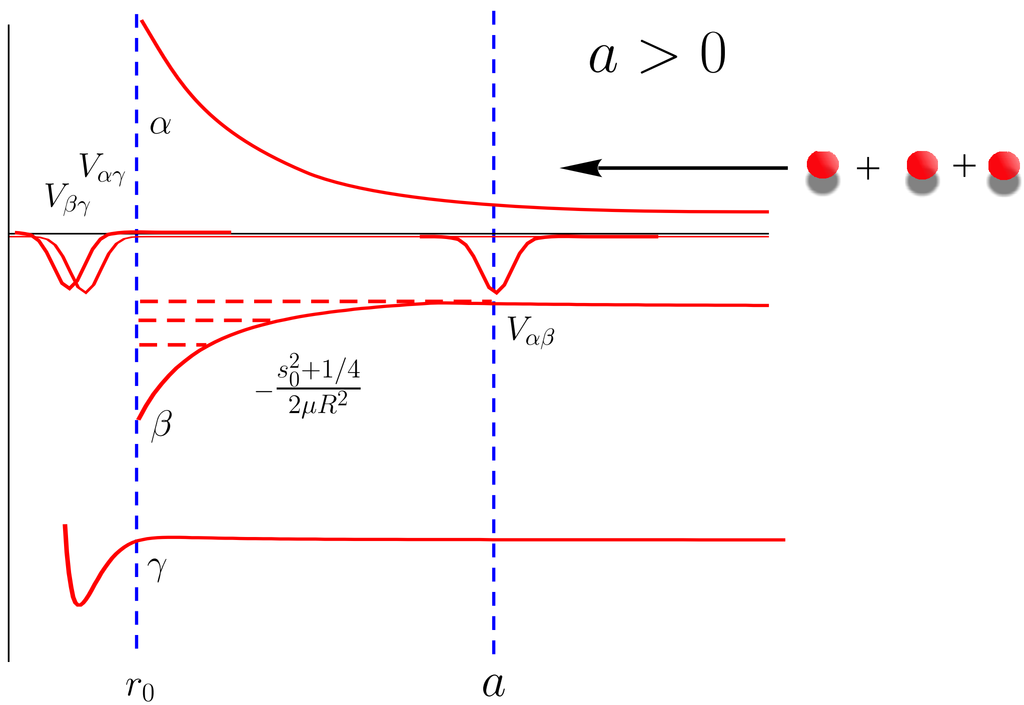


Signatures of The Efimov Effect

Three-Body Recombination





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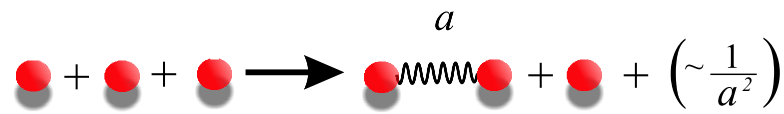

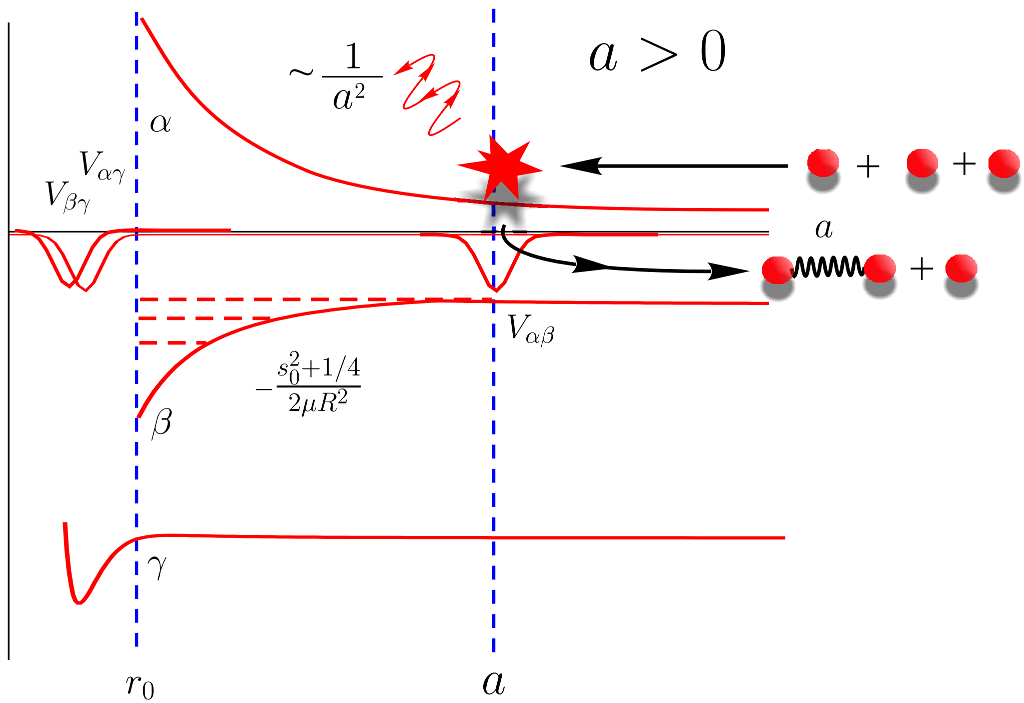

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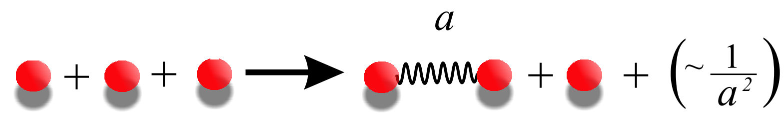

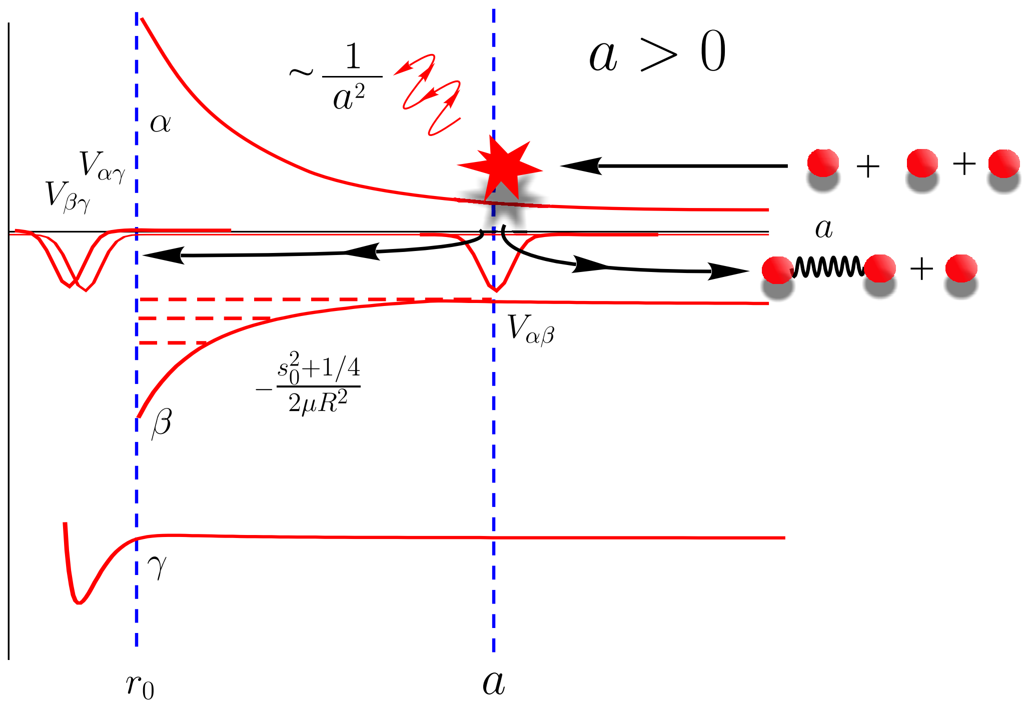

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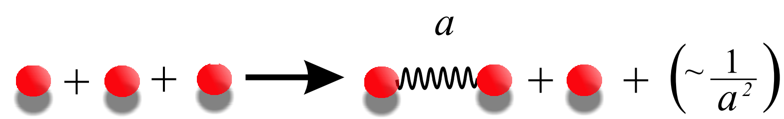

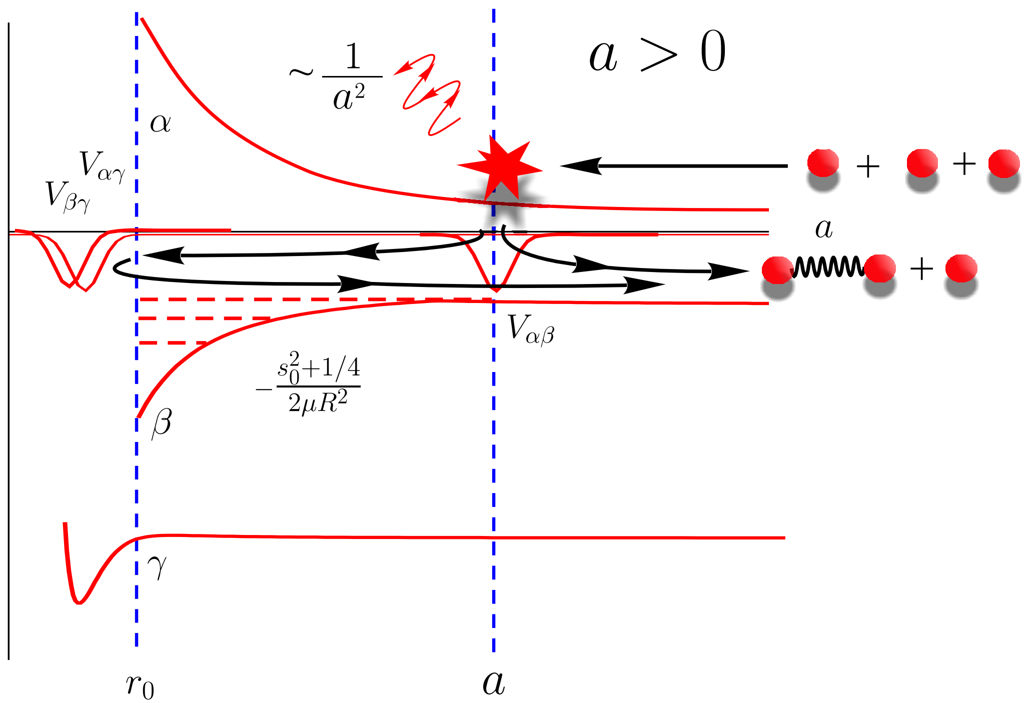

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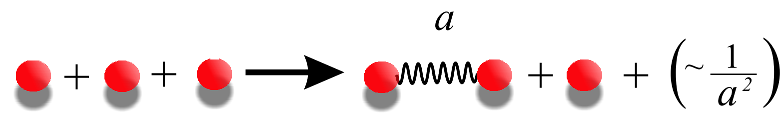

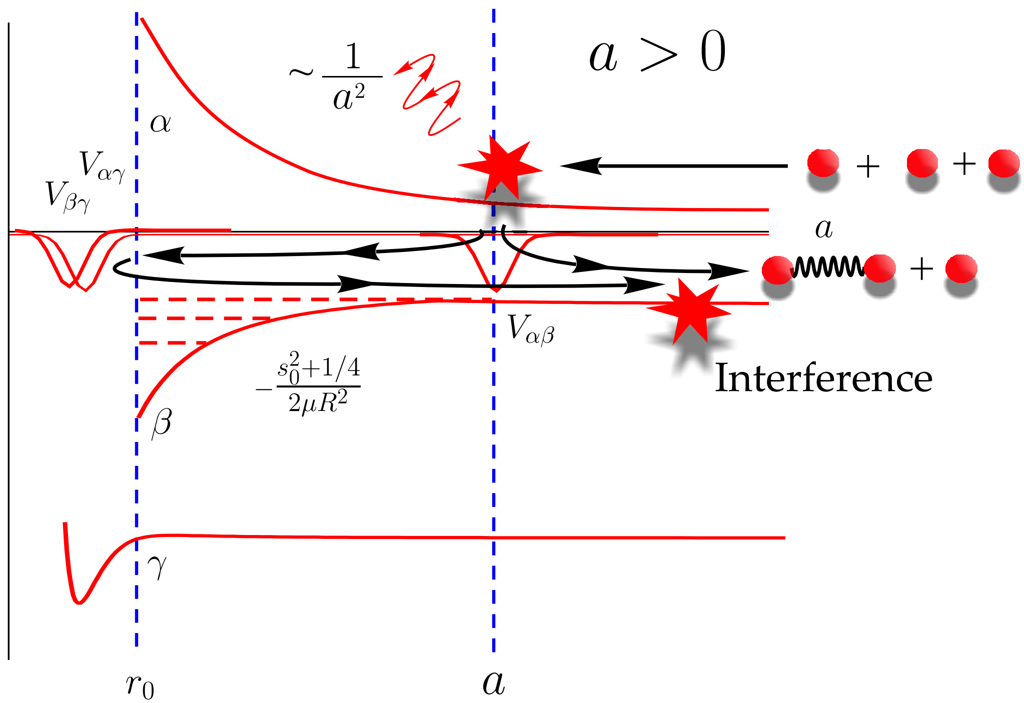

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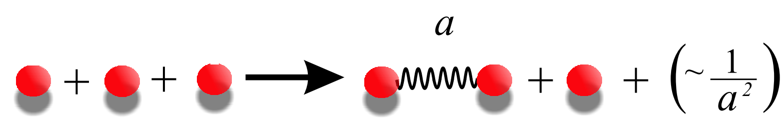

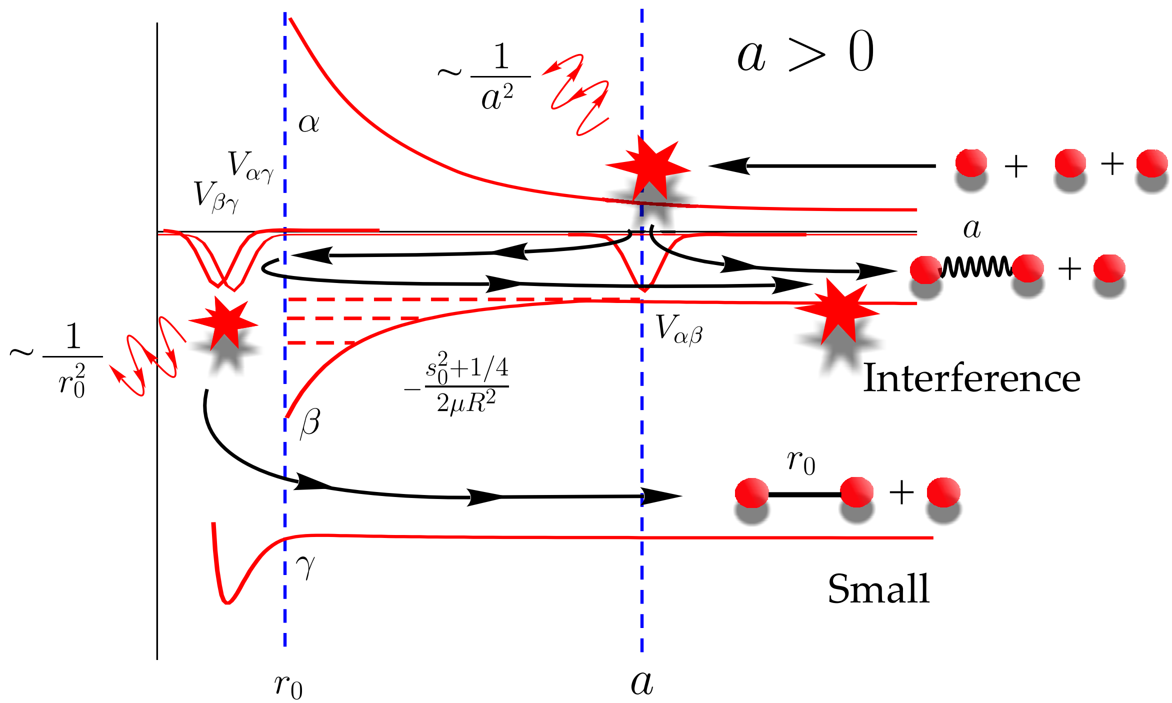

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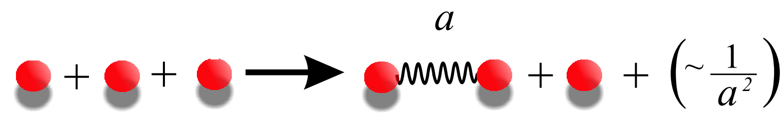
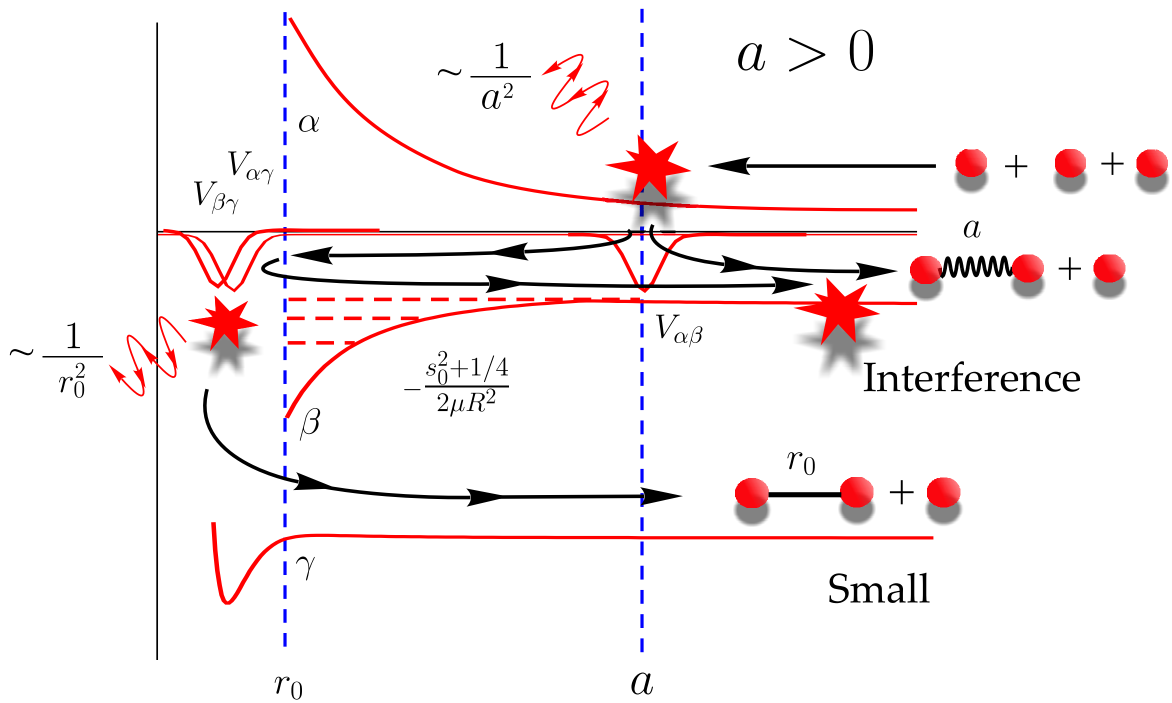


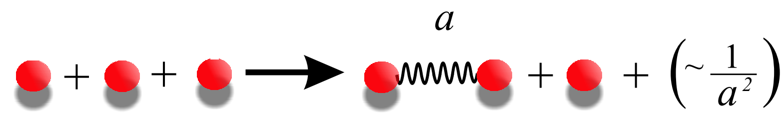


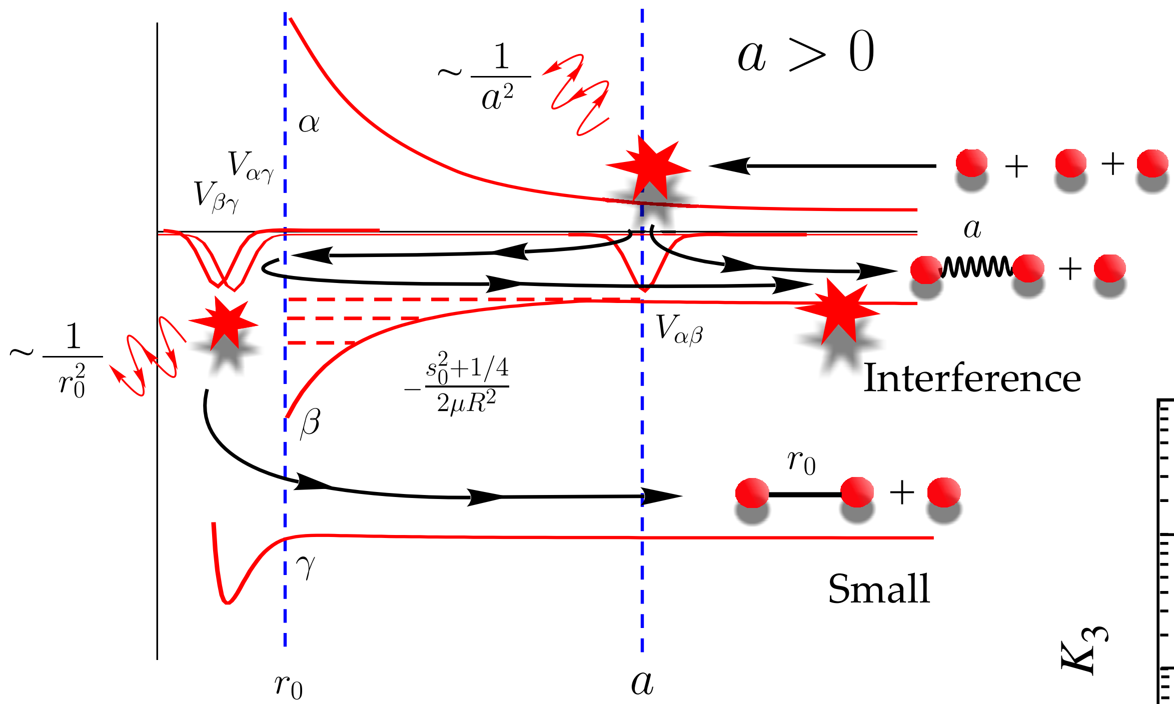
$$K_3 = B_\eta \sin^2\left[s_0 \ln\left(\frac{a}{r_0}\right) + \Phi\right] a^4$$

B_η, Φ: details !

Signatures of The Efimov Effect

Three-Body Recombination





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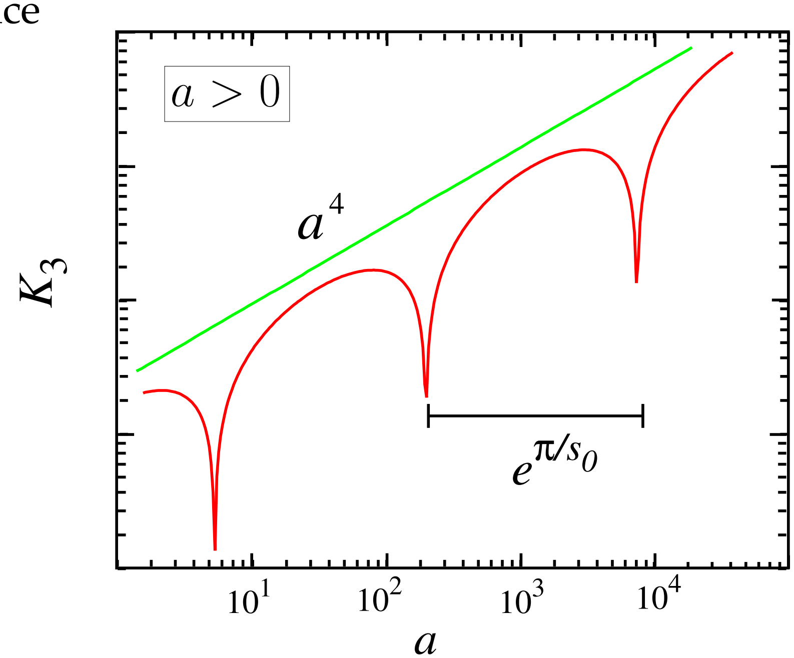
B_η, Φ: details !

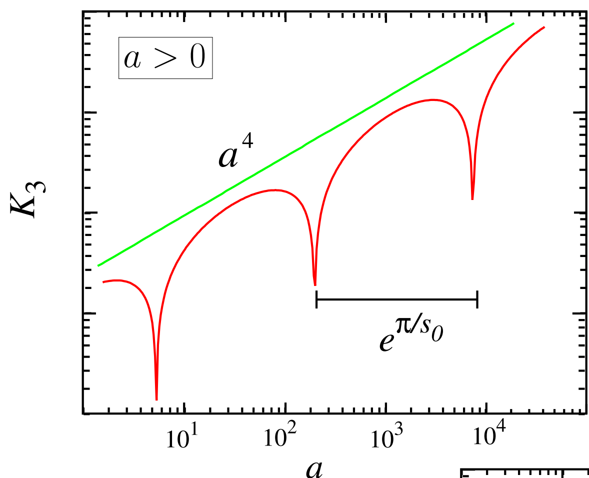


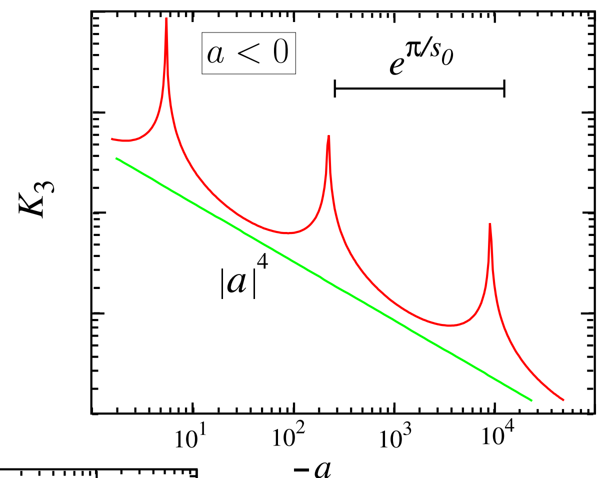
OBSERVATION OF THE EFIMOV EFFECT

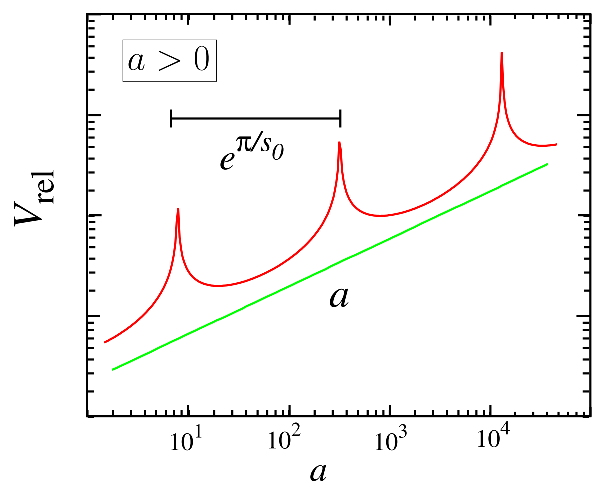
Rate Equations:

$$\dot{n}_X(t) = -[K_3(a)]n_X^3 - [V_{\rm rel}(a)]n_X n_{X_2}$$

$$\dot{n}_{X_2}(t) = -[V_{\rm rel}(a)]n_X n_{X_2}$$

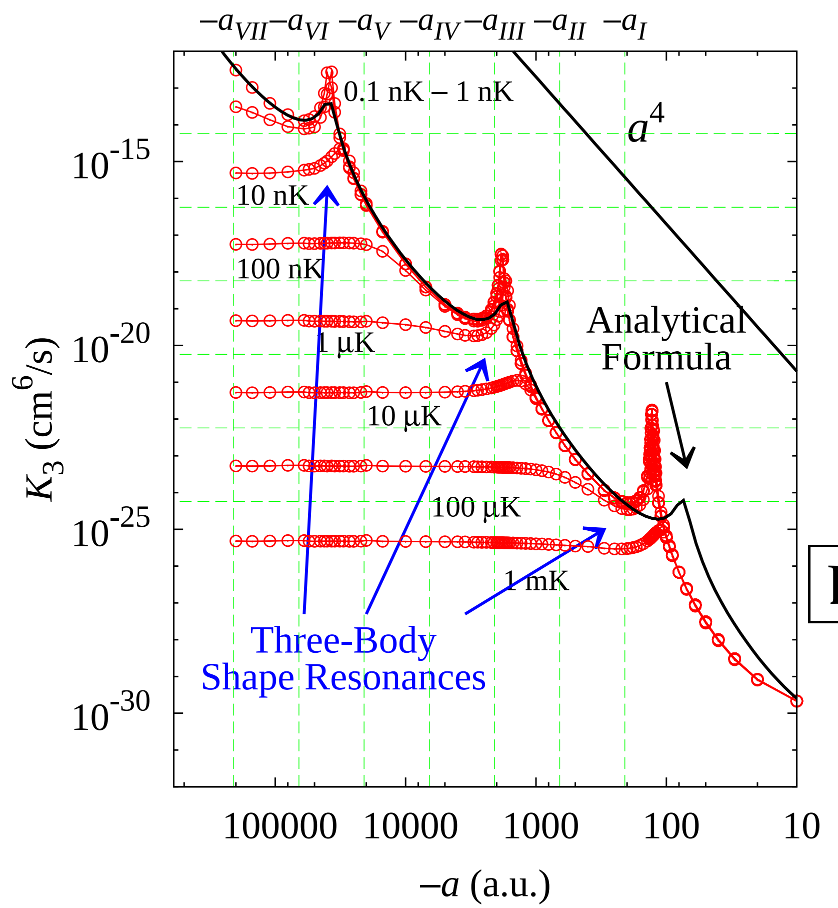






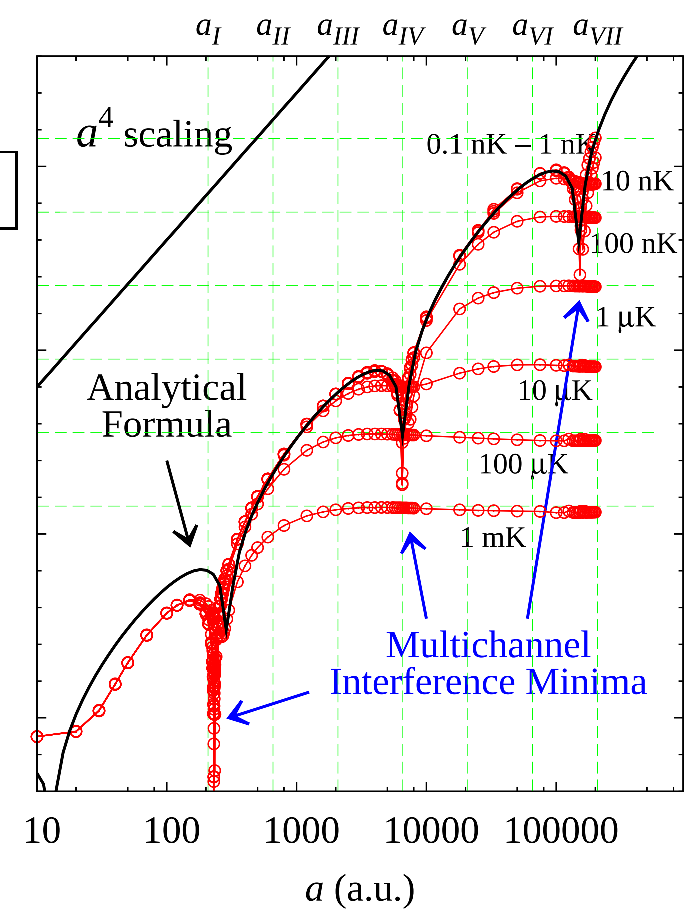
Three features need to be observed !

LIMITATIONS



At finite temperatures only a finte number of features can be expected to be observed.

B+B+B Collisions



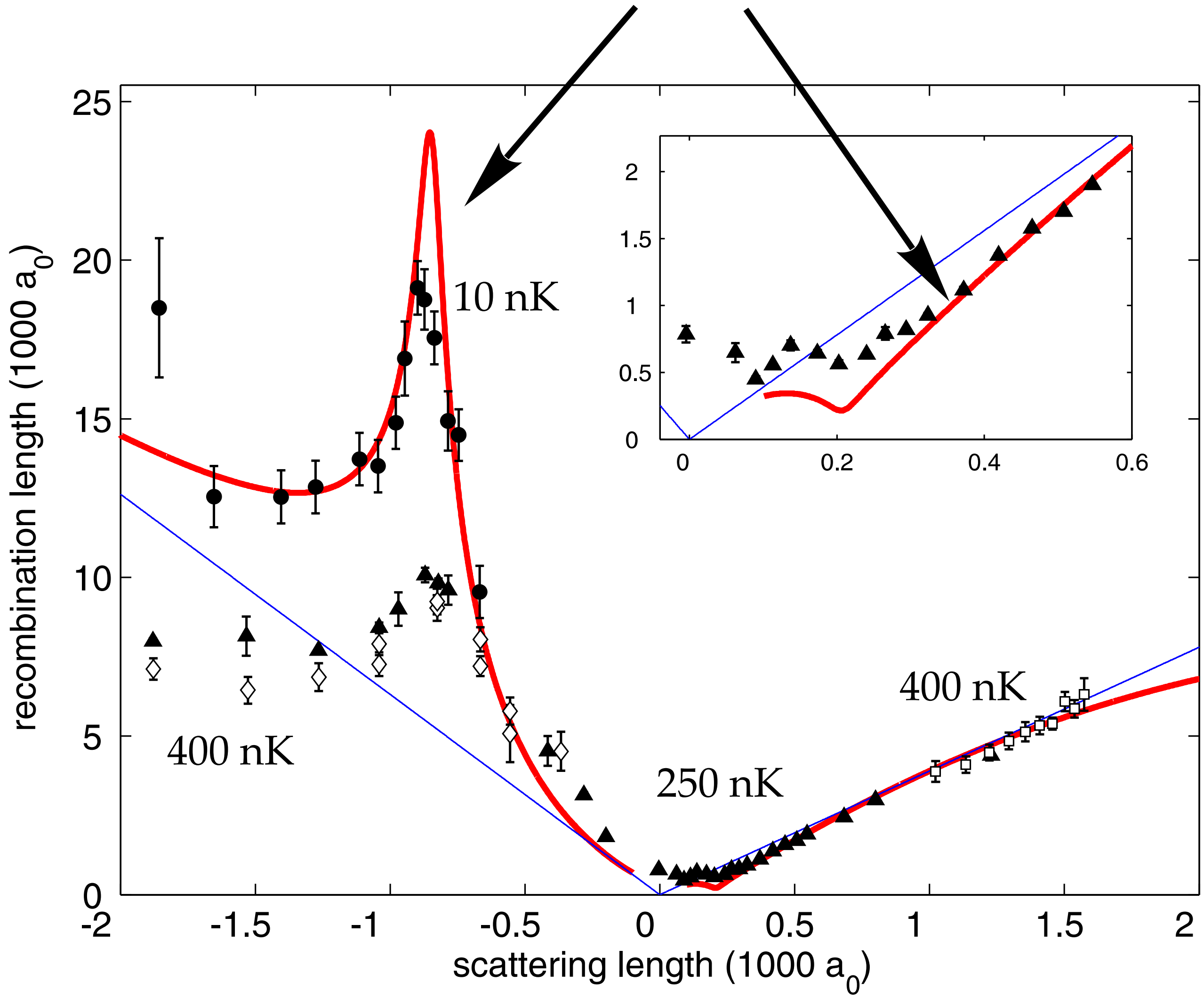
$$a_{\min} \approx r_0[e^{\pi/s_0}]^N \Rightarrow T_{\max} \lesssim 1/ma^2_{\min}$$

(spacing: $e^{\pi/s_0} \approx 22.7$)

Large a / Low T !

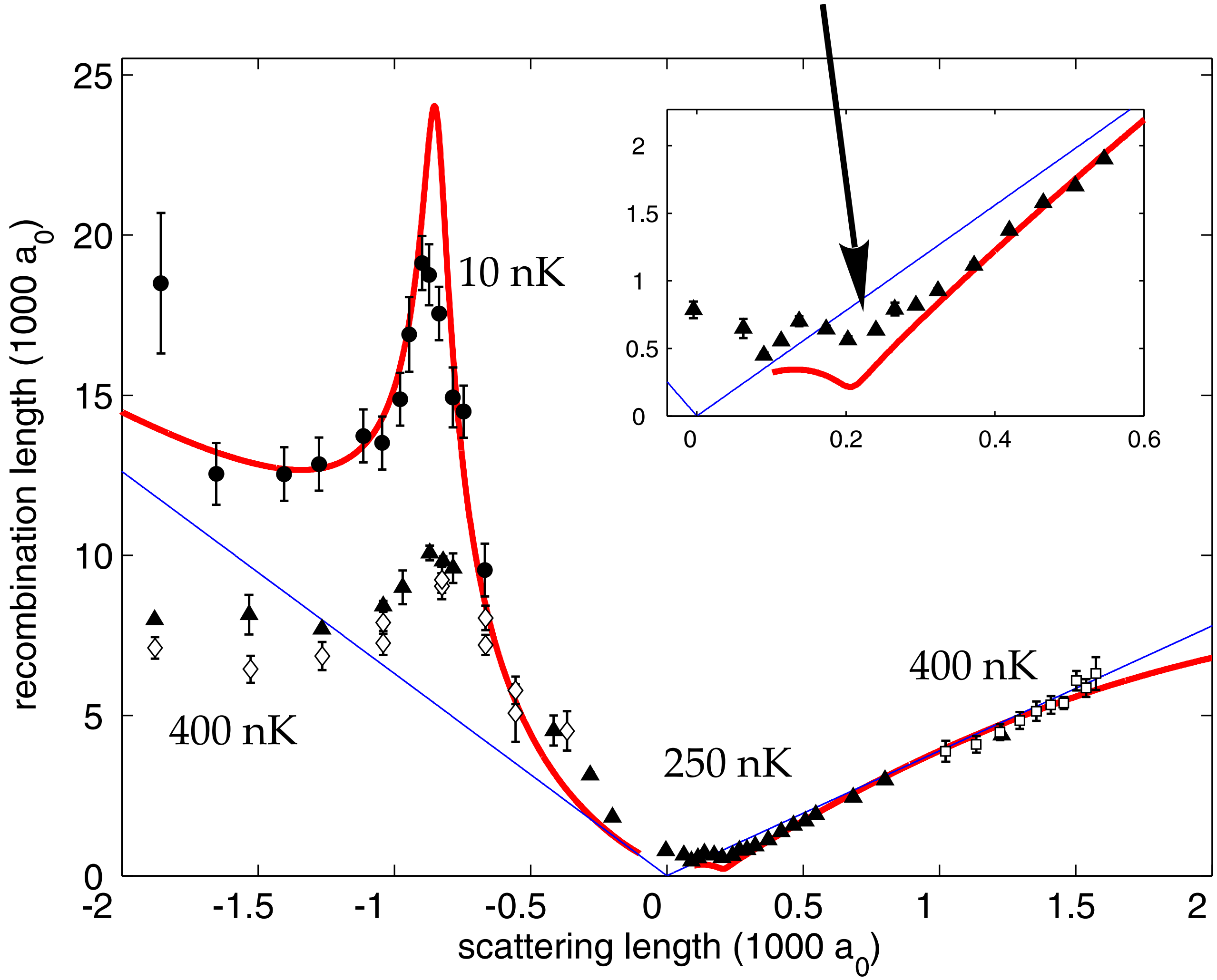
Cesium Experiment - Innsbruck
Rudi Grimm's Group

Analytical Formula

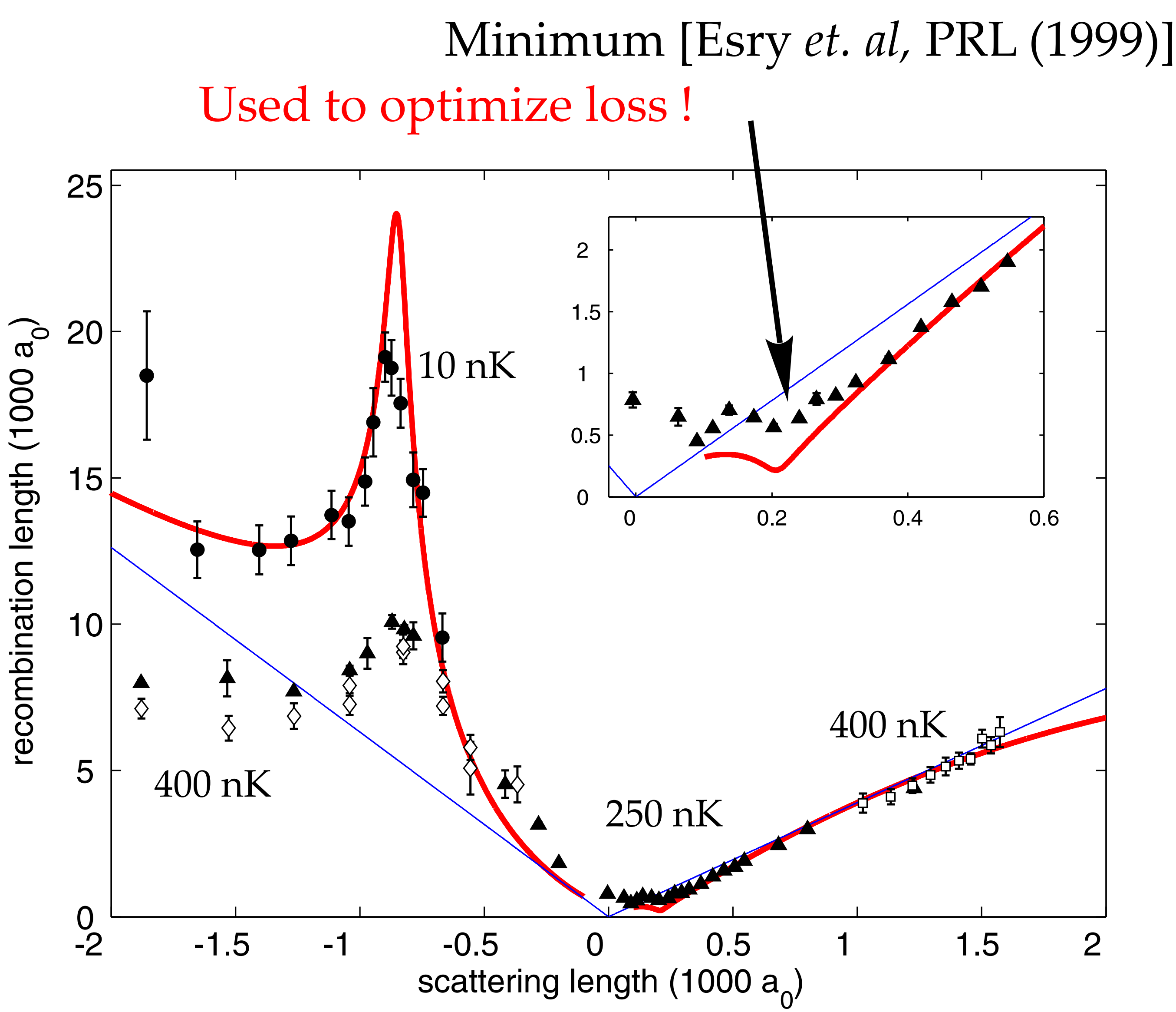


Cesium Experiment - Innsbruck
Rudi Grimm's Group

Minimum [Esry *et. al,* PRL (1999)]

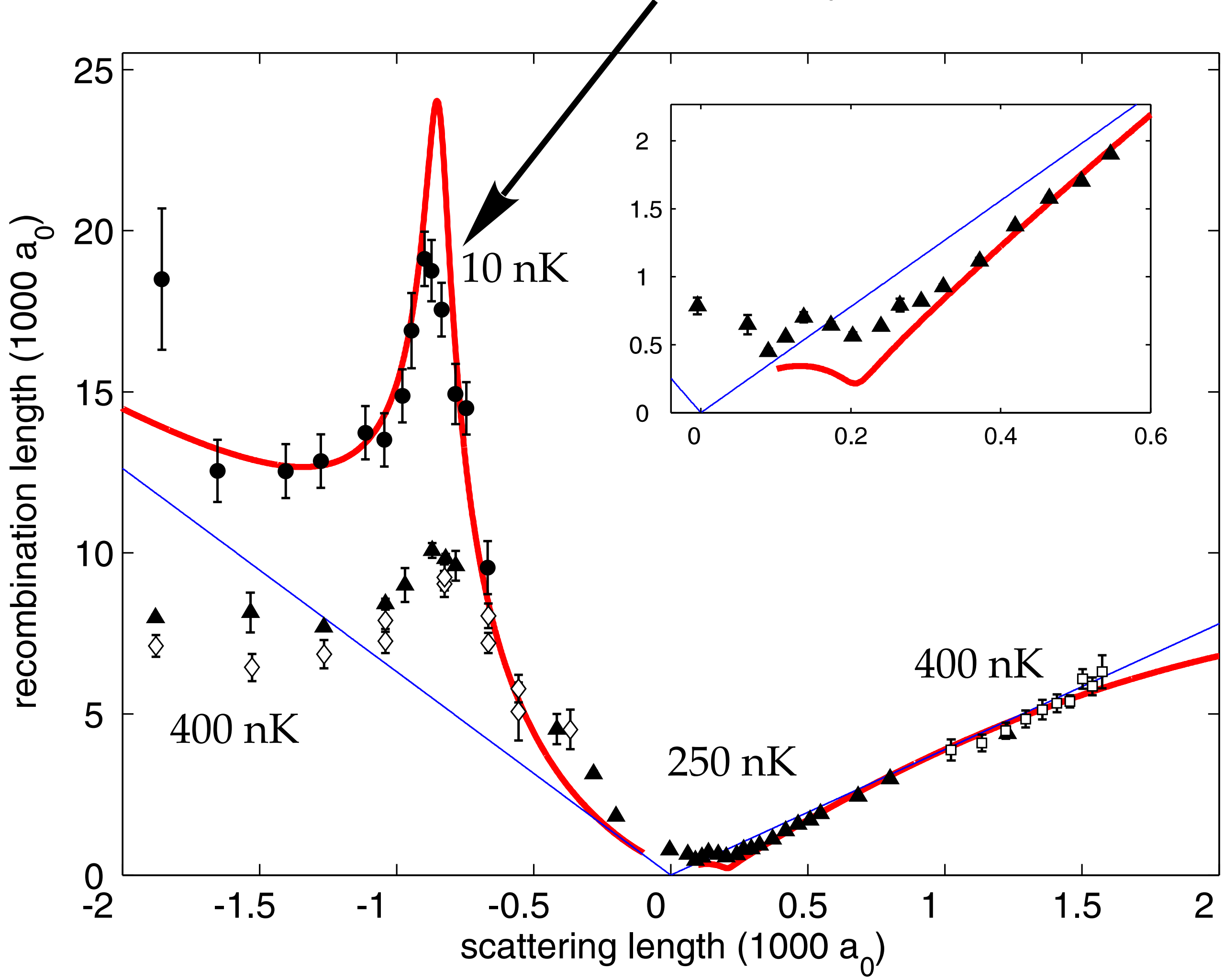


CESIUM EXPERIMENT - INNSBRUCK
RUDI GRIMM'S GROUP



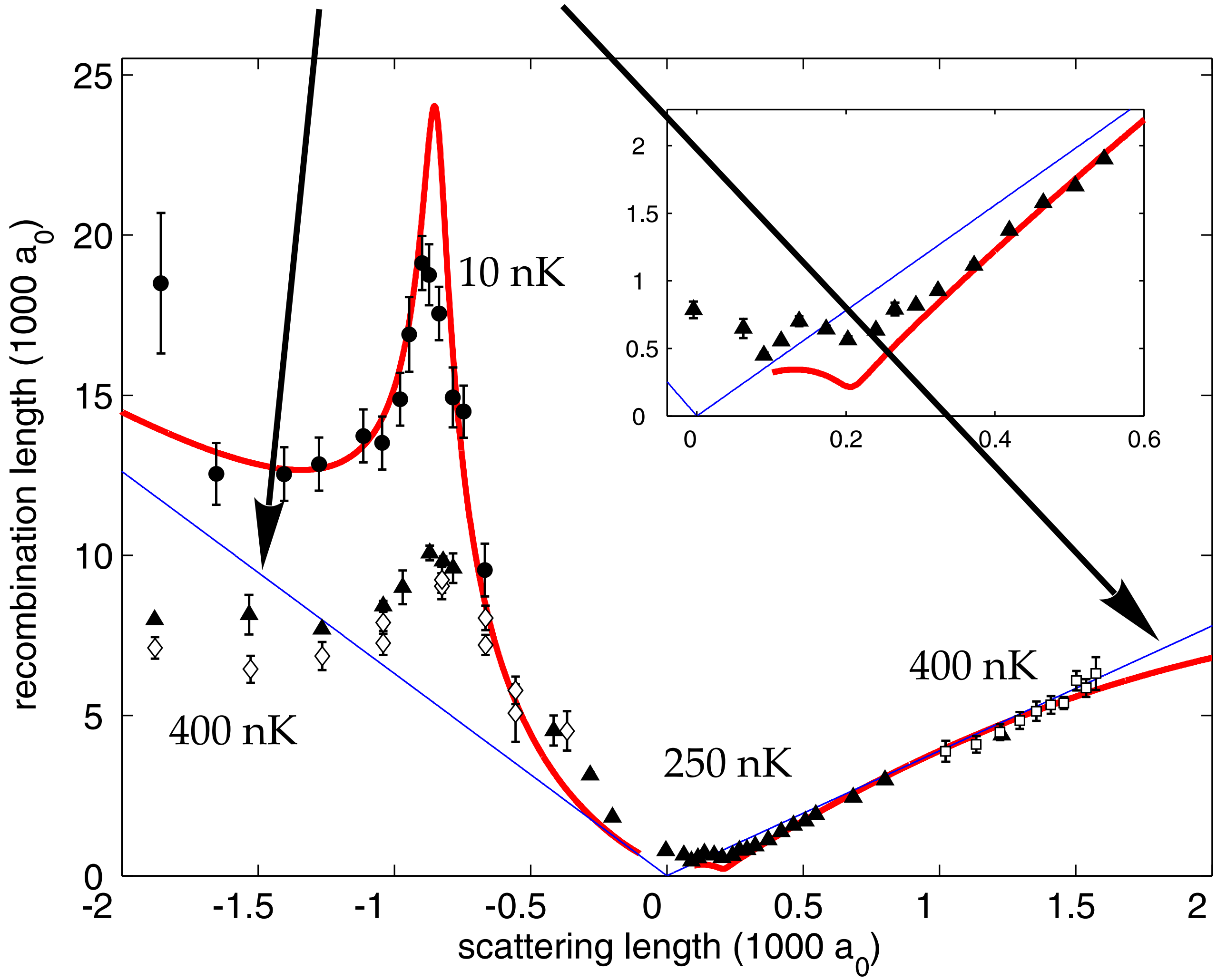
CESIUM EXPERIMENT - INNSBRUCK
RUDI GRIMM'S GROUP

Peak [Esry *et. al,* PRL (1999)]



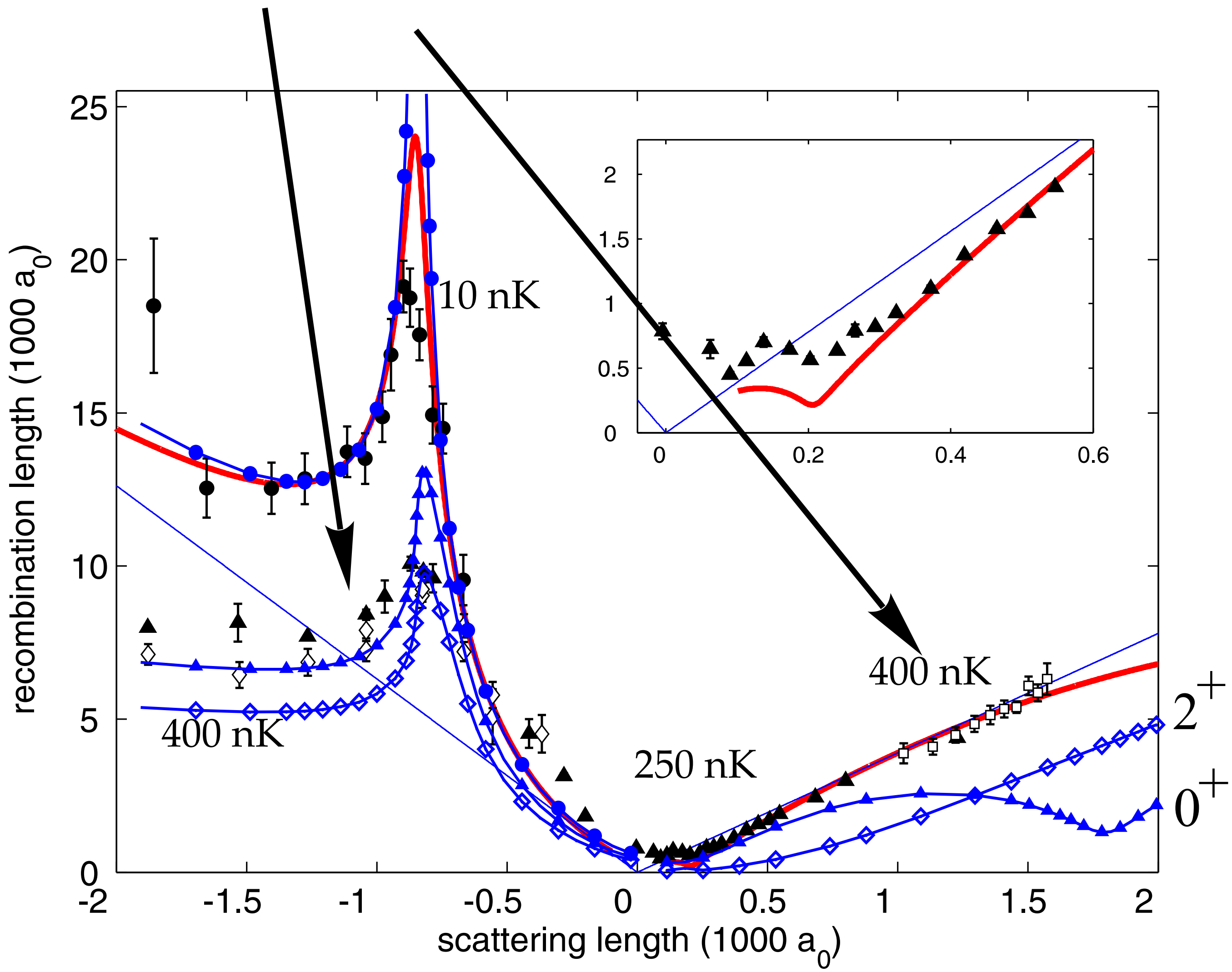
Cesium Experiment - Innsbruck
Rudi Grimm's Group

Different slopes [Esry *et. al*, PRL (1999)]



CESIUM EXPERIMENT - INNSBRUCK
RUDI GRIMM'S GROUP

Finite Energy Effects [D'Incao, Suno, Esry, PRL (2004)]



Single specie Atomic Gases

- Large spacing ($e^{\pi/s_0} \approx 22.7$) between Efimov features
- $a_{\rm min} \approx r_0[e^{\pi/s_0}]^N \to T_{\rm max} \lesssim 1/ma^2_{\rm min}$: large a, low T
- ^{133}Cs: $a_{\rm min} \approx 4. \times 10^6$ a.u.; $T_{\rm max} \approx 8. \times 10^{-5}$ nK (✗)

Two species Atomic Gases

- Spacing (e^{π/s_0}) can be made smaller (✓)
- Experimentally accessible $a_{\rm min}$ and $T_{\rm max}$ (✓)
- Competition different 3-body processes: important (?)
- Favorable conditions: Boson-Fermion mixtures (✓)

TWO SPECIES ATOMIC GASES

EXPERIMENTS:

^{87}Rb-^{40}K (JILA)

^{23}Na-^{6}Li (MIT) Interspecies Feshbach resonances !

Two types of collisions are important: XXY and XYY

Recombination (no molecules !)

$$\dot{n}_{\rm X} \approx -[K_3^{\rm X+X+Y}(a)]n_{\rm Y}n_{\rm X}^2 - [K_3^{\rm X+Y+Y}(a)]n_{\rm Y}^2 n_{\rm X}$$

$$\dot{n}_{\rm Y} \approx -[K_3^{\rm X+X+Y}(a)]n_{\rm X}^2 n_{\rm Y} - [K_3^{\rm X+Y+Y}(a)]n_{\rm X}n_{\rm Y}^2,$$

Relaxation

Competition !

$$\dot{n}_{\rm X} \approx -[V_{\rm rel}^{\rm XY+X}(a)]n_{\rm XY}n_{\rm X}$$

$$\dot{n}_{\rm Y} \approx -[V_{\rm rel}^{\rm XY+Y}(a)]n_{\rm XY}n_{\rm Y}$$

$$\dot{n}_{\rm XY} \approx -[V_{\rm rel}^{\rm XY+X}(a)]n_{\rm X}n_{\rm XY} - [V_{\rm rel}^{\rm XY+Y}(a)]n_{\rm Y}n_{\rm XY}$$

Two Species Atomic Gases

Efimov Physics $\longrightarrow a$ dependence in K_3, $V_{\rm rel}$.

[Gross scaling, valid for $E \lesssim 1/2\mu a^2$]

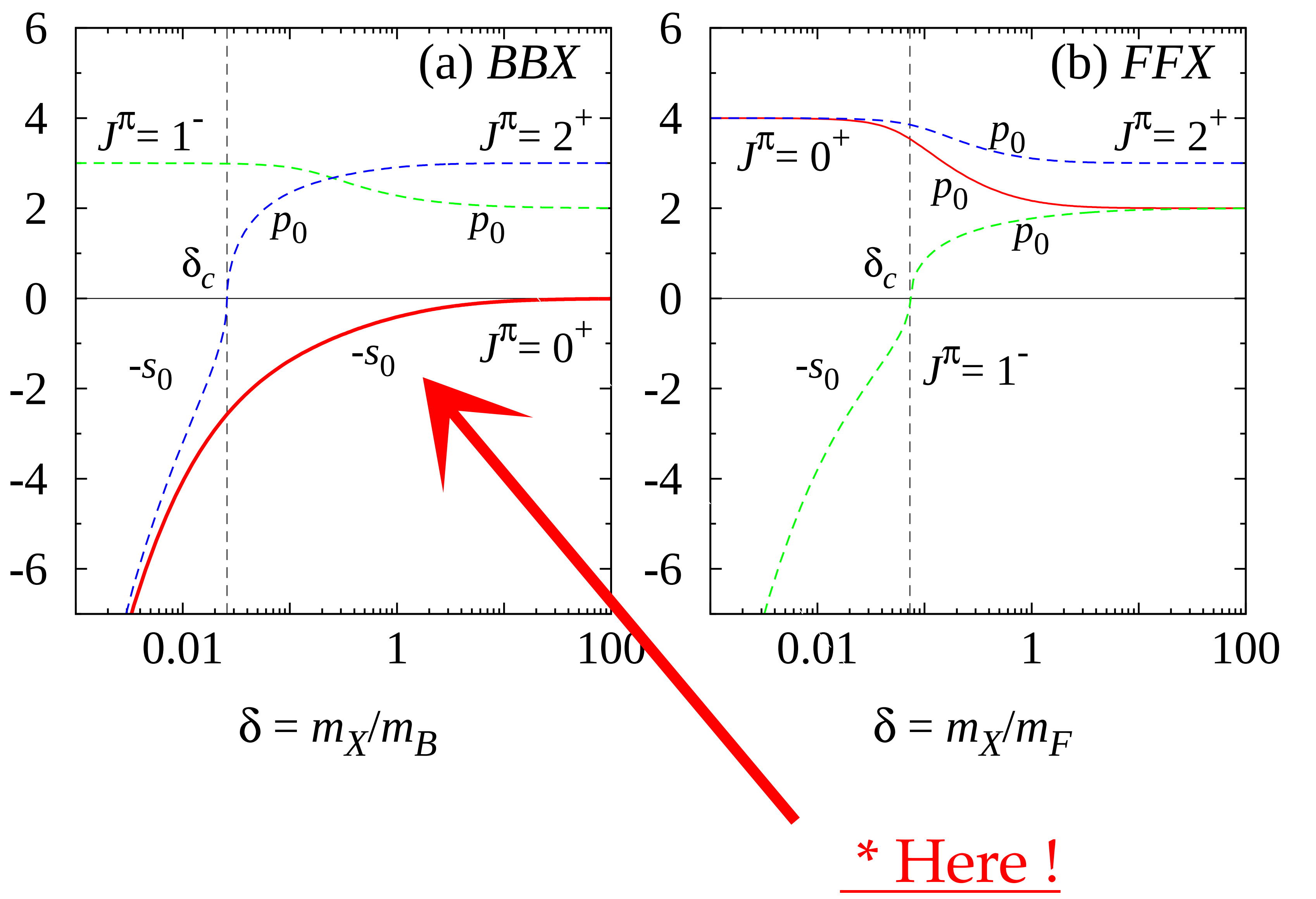
	J^π	$V_{\rm rel}$: E	$V_{\rm rel}$: $a>0$	$V_{\rm rel}$: $a<0$	K_3 (D_3): E	K_3 (D_3): $a>0$	K_3 (D_3): $a<0$
BBX	0^{+*}	**const**	$*[P_{s_0}]a$	**const**	**const**(k^4)	$*[M_{s_0}]a^4$	$*[P_{s_0}]\|a\|^4$
$\delta=\frac{m_X}{m_B}$	1^-	k^2	a^{3-2p_0}	const	$k^2(k^6)$	a^6	$\|a\|^{6-2p_0}$
	$2^{+}_{\delta<\delta_c}{}^*$	k^4	$*[P_{s_0}]a^5$	const	$k^4(k^8)$	$*[M_{s_0}]a^8$	$*[P_{s_0}]\|a\|^8$
	$2^{+}_{\delta>\delta_c}$	k^4	a^{5-2p_0}	const	$k^4(k^8)$	a^8	$\|a\|^{8-2p_0}$
FFX	0^+	**const**	a^{1-2p_0}	**const**	$k^4(k^8)$	a^8	$\|a\|^{8-2p_0}$
$\delta=\frac{m_X}{m_F}$	$1^{-}_{\delta<\delta_c}{}^*$	k^2	$*[P_{s_0}]a^3$	const	$k^2(k^6)$	$*[M_{s_0}]a^6$	$*[P_{s_0}]\|a\|^6$
	$1^{-}_{\delta>\delta_c}$	k^2	a^{3-2p_0}	const	$k^2(k^6)$	a^6	$\|a\|^{6-2p_0}$
	2^+	k^4	a^{5-2p_0}	const	$k^4(k^8)$	a^8	$\|a\|^{8-2p_0}$

D'Incao and Esry, Submitted to PRL.

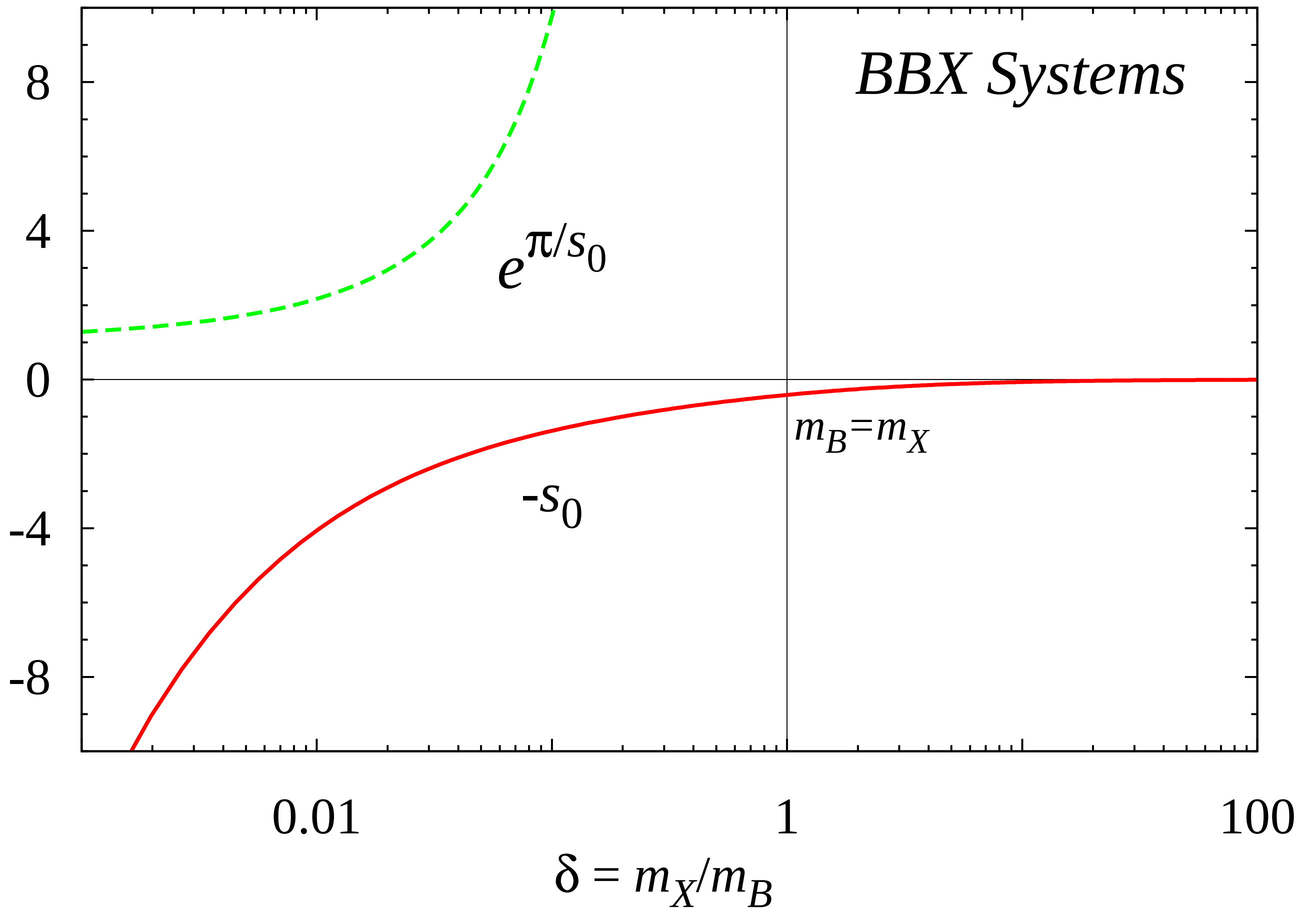
(*) Efimov Effect !

s_0 depends on δ!

Two Species Atomic Gases



Two Species Atomic Gases



We want mixtures with heavy Bosons ($m_B >> m_X$) !

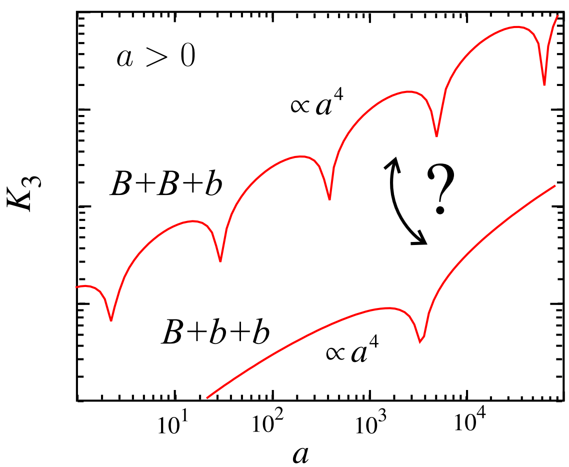
Efimov Effect in Boson-Boson Mixtures

$m_B \gg m_b$	$B\,B\,b$ collisions: s_0 large $\rightarrow e^{\pi/s_0}$ small (✓) $B\,b\,b$ collisions: s_0 small $\rightarrow e^{\pi/s_0}$ large (✗)

Efimov Effect in Boson-Boson Mixtures

$m_B \gg m_b$	$B\,B\,b$ collisions: s_0 large $\rightarrow e^{\pi/s_0}$ small (✓) $B\,b\,b$ collisions: s_0 small $\rightarrow e^{\pi/s_0}$ large (✗)

Three-Body Recombination

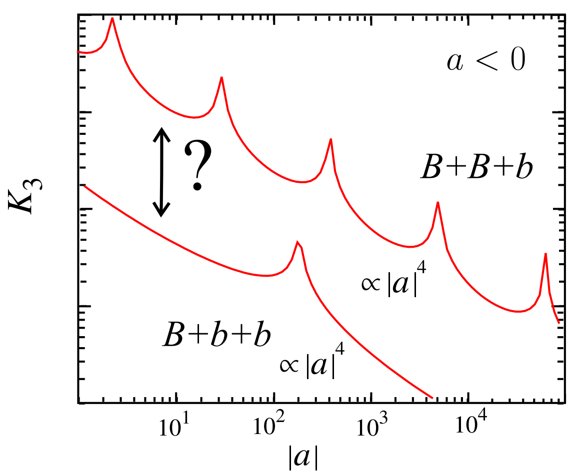


Condition: $n_\mathrm{B} \gg n_\mathrm{b}$

$$\dot{n}_\mathrm{B} = -K_3^{B+B+b} n_\mathrm{b} {n_\mathrm{B}}^2$$

$$\dot{n}_\mathrm{b} = -K_3^{B+B+b} n_\mathrm{B}^2 n_\mathrm{b}$$

enough signal ? (✗)



Efimov Effect in Boson-Boson Mixtures

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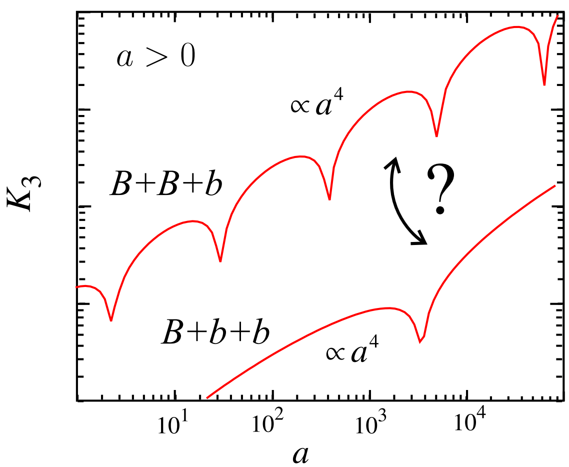
Three-Body Recombination

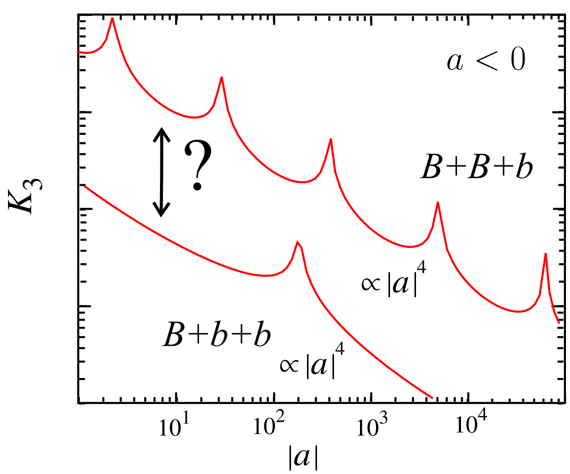


Condition: $n_\mathrm{B} \gg n_\mathrm{b}$

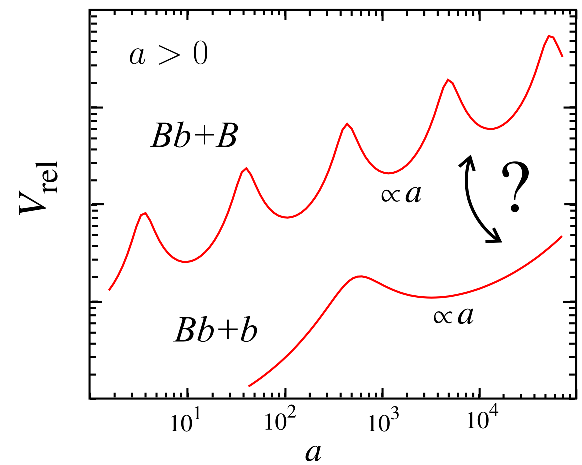
$$\dot{n}_\mathrm{B} = -K_3^{B+B+b} n_\mathrm{b} n_\mathrm{B}{}^2$$

$$\dot{n}_\mathrm{b} = -K_3^{B+B+b} n_\mathrm{B}^2 n_\mathrm{b}$$

enough signal ? (✗)



Vibrational Relaxation



Condition: $n_\mathrm{b} = 0$

$$\dot{n}_\mathrm{B} = -V_\mathrm{rel}^{Bb+B} n_\mathrm{B} n_\mathrm{Bb}$$

$$\dot{n}_\mathrm{Bb} = -V_\mathrm{rel}^{Bb+B} n_\mathrm{Bb} n_\mathrm{B}$$

$Bb + Bb$ collisions can be important ! (✗)

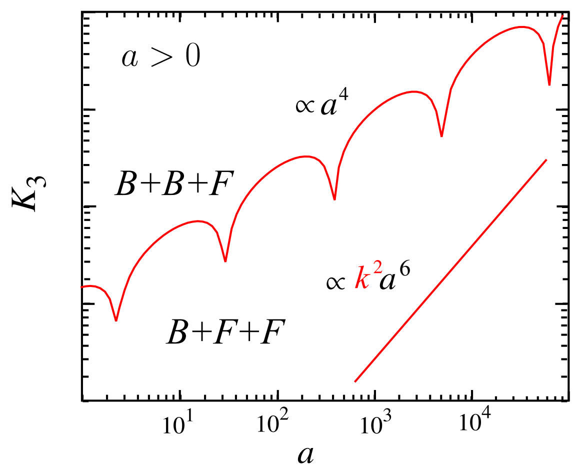
Efimov Effect in Boson-Fermion Mixtures

$m_B \gg m_F$	BBF collisions: s_0 large $\rightarrow e^{\pi/s_0}$ small (✓) BFF collisions: no Efimov Effect (✗)

Efimov Effect in Boson-Fermion Mixtures

$m_B \gg m_F$	BBF collisions: s_0 large $\rightarrow e^{\pi/s_0}$ small (✓) BFF collisions: no Efimov Effect (✗)

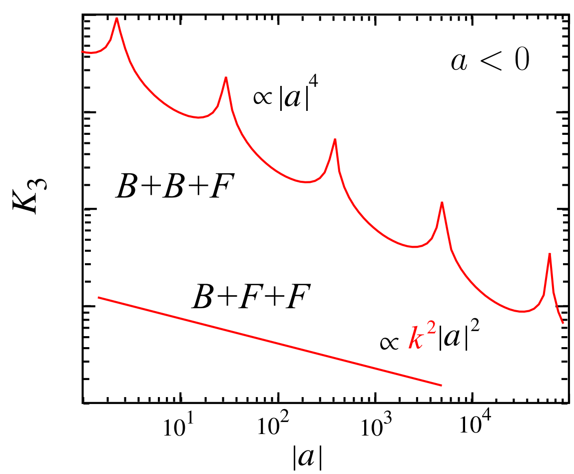
Three-Body Recombination



Condition: $T \lesssim T_{\max}$ (✓)

$$\dot{n}_{\mathrm{B}} = -K_3^{B+B+F} n_{\mathrm{F}} {n_{\mathrm{B}}}^2$$

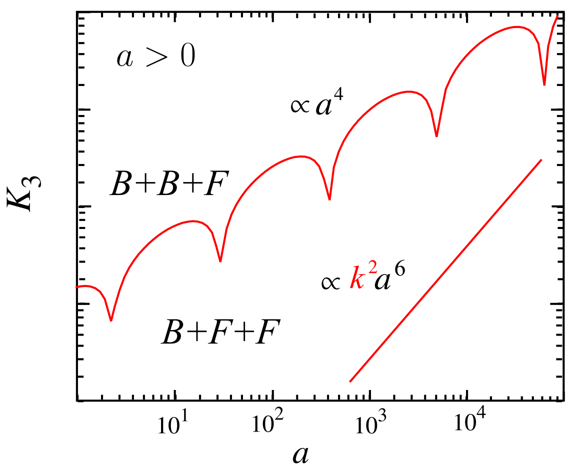
$$\dot{n}_{\mathrm{F}} = -K_3^{B+B+F} n_{\mathrm{B}}^2 n_{\mathrm{F}}$$



Efimov Effect in Boson-Fermion Mixtures

$m_B \gg m_F$	BBF collisions: s_0 large $\to e^{\pi/s_0}$ small (✓) BFF collisions: no Efimov Effect (✗)

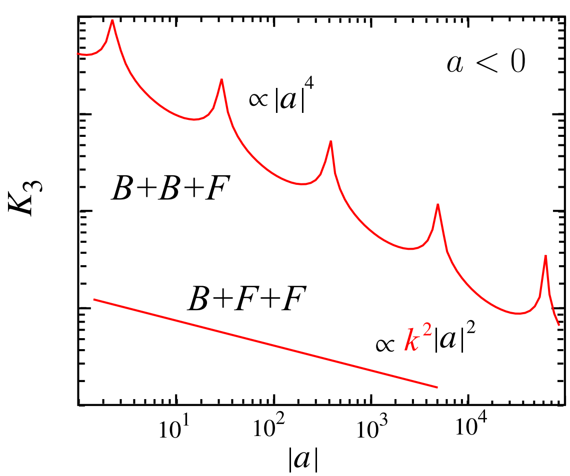
Three-Body Recombination



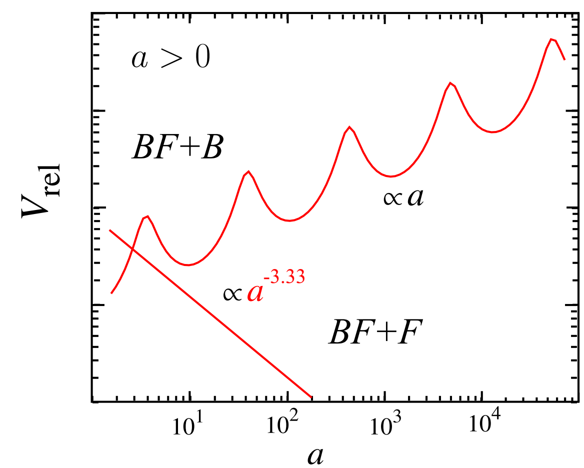
Condition: $T \lesssim T_{\rm max}$ (✓)

$$\dot{n}_{\rm B} = -K_3^{B+B+F} n_{\rm F} {n_{\rm B}}^2$$

$$\dot{n}_{\rm F} = -K_3^{B+B+F} n_{\rm B}^2 n_{\rm F}$$



Vibrational Relaxation



Condition: $T \lesssim T_{\rm max}$ (✓)

$$\dot{n}_{\rm B} = -V_{\rm rel}^{BF+B} n_{\rm B} n_{\rm BF}$$

$$\dot{n}_{\rm BF} = -V_{\rm rel}^{BF+B} n_{\rm BF} n_{\rm B}$$

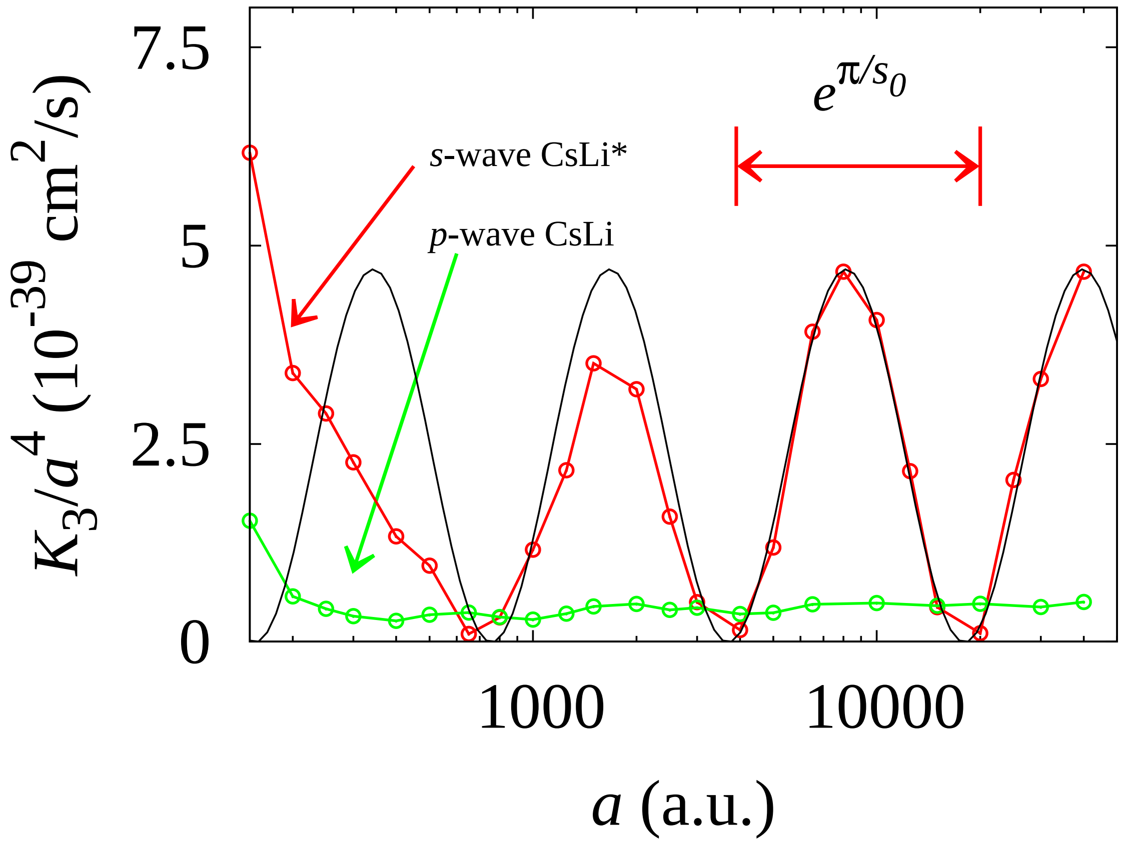
$BF + BF$ collisions suppressed (p-wave)

Boson-Fermion Mixtures

	K_3^{B+B+F} and $V_{\rm rel}^{BF+B}$			
$B-F$	e^{π/s_0}	$\lvert a_{\rm min}\rvert$(a.u.)	$T_{\rm max}$(nK)	
^{133}Cs–^{6}Li	4.877	1.6×10^4	60.0	✓
^{87}Rb–^{6}Li	6.856	5.6×10^4	5.00	✓
^{23}Na–^{6}Li	36.28	3.3×10^8	$\ll 0.1$	✗ (MIT)
^{7}Li–^{6}Li	>100	$\gg 10^8$	$\ll 0.1$	✗
^{133}Cs–^{40}K	47.02	9.2×10^7	$\ll 0.1$	✗
^{87}Rb–^{40}K	>100	$\gg 10^8$	$\ll 0.1$	✗ (JILA)
^{23}Na–^{40}K	>100	$\gg 10^8$	$\ll 0.1$	✗
^{7}Li–^{40}K	>100	$\gg 10^8$	$\ll 0.1$	✗

BOSON-FERMION MIXTURES

$^{133}Cs + ^{133}Cs + ^{6}Li$



SUMMARY

- Ultracold Quantum Gases: clear signature of EFIMOV EFFECT

- Boson-Fermion mixtures ($m_B \gg m_F$): favorable system

- Extremely long-lived BF molecules: EFIMOV PHYSICS